Phys 207 Exam #2 (4/21/05) **"Alphabetic" Student Number:** **Key #1**

Overall Exam Average = <u>76.8%</u> (19% A's, 28% B's, 28% C's, 10% D's, 15% F's)

For multiple choice answers, the correct answer is highlighted. **(Average on all multiple choice = 64%** AFTER adding 3 pts for low-scoring question #5; Original average = 58%**)**

1. If a **net force** is exerted on an object **along** its direction of motion, then (**ave = 76%**)
   (a) work may be done and momentum always changes.
   (b) work may be done and momentum may change.
   (c) work is always done and momentum always changes.
   (d) work is always done and momentum may change.

2. A box is vertically dropped from rest. What is the relationship between the **speed** $\boldsymbol{v_1}$ after the box has fallen a **height** $\boldsymbol{h}$ and the **speed** $\boldsymbol{v_2}$ after the box has fallen a **height 4*h***? (**ave = 59%**)
   (a) $v_2 = v_1$
   (b) $v_2 = \sqrt{2}v_1$
   (c) $\boxed{v_2 = 2v_1}$ $\quad \Delta K = -\Delta U \Rightarrow \frac{1}{2}mv^2 = mgh \quad \boxed{v} = \sqrt{2gh}$ and $\boxed{\frac{v_2}{v_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{4} = 2$
   (d) $v_2 = 4v_1$
   (e) None of the above.

3. Ball #1 (**mass *M***) and ball #2 (**mass 2*M***) are dropped simultaneously from the same height above the ground. At any given **instant in time** when the balls are falling, what is the relationship between the **impulse** $\boldsymbol{I_1}$ on ball #1 and **impulse** $\boldsymbol{I_2}$ on ball #2? (**ave = 71%**)
   (a) $I_2 = 4I_1$
   (b) $\boxed{I_2 = 2I_1}$ $\quad \boxed{I} = \vec{F}\Delta t = (mg)\Delta t$ and $\frac{I_2}{I_1} = \frac{m_2}{m_1} = 2$ since Δt same
   (c) $I_2 = I_1$
   (d) $I_2 = \frac{1}{2}I_1$
   (e) None of the above.

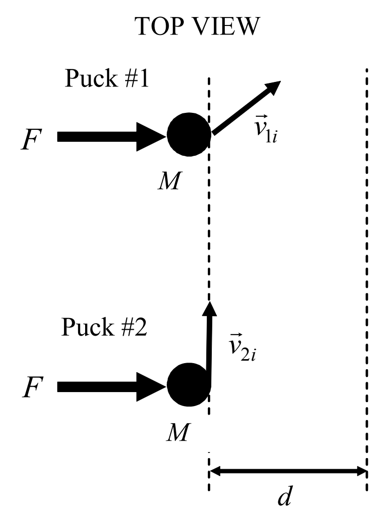
4. On a frictionless surface, a box is pushed against a fixed spring and compresses it a **distance** $\boldsymbol{x_1}$ from its relaxed position. When the box is released, it slides away from the spring with **velocity** $\boldsymbol{v_1}$. If the spring were **compressed to twice the distance** (i.e., $x_2 = 2x_1$), then what would be the new **velocity** $\boldsymbol{v_2}$ of the box? (**ave = 72%**)
   (a) $v_2 = v_1$
   (b) $v_2 = \sqrt{2}v_1$
   (c) $\boxed{v_2 = 2v_1}$ $\quad \frac{1}{2}kx^2 = \frac{1}{2}m\,v^2 \Rightarrow \frac{v_2}{v_1} = \frac{x_2}{x_1} = 2$
   (d) $v_2 = 4v_1$
   (e) None of the above.

5. Cart #1 travels with momentum **+10 kg m/s** and hits a **stationary** Cart #2. After the collision, cart #1 travels with momentum **–5 kg m/s** and cart #2 travels with momentum **+15 kg m/s**. The collision type and relative masses of the carts are: (**ave = 16%**)
   (a) Elastic, Cart #1 heavier
   (b) Elastic, Cart #2 heavier
   (c) Inelastic, Cart #1 heavier
   (d) Inelastic, Cart #2 heavier
   (e) Insufficient information Not possible to determine kinetic energy with info given.

6. A heavier box and a lighter box are both at rest on a frictionless surface. The same force F pushes on each box for a **distance of 1 m**. Which box has the **larger final momentum** after the force acts? (**ave = 62%**)
   (a) Lighter box
   (b) Same
   (c) Heavier box Larger impulse since it requires a longer time to travel 1 m.
   (d) Insufficient information

7. When two particles on a horizontal frictionless surface collide and **stick together**, then (**ave = 79%**)
   (a) Momentum is not conserved and kinetic energy is not conserved.
   (b) Momentum is not conserved and kinetic energy is conserved.
   (c) Momentum is conserved and kinetic energy is conserved.
   (d) Momentum is conserved and kinetic energy is not conserved.

Puck #1 (**mass *M***) and puck #2 (**mass *M***) slide across a horizontal, frictionless table with **identical speeds**, but in **different directions**. The **same force *F*** is exerted on each puck as it travels from the first to the second dotted line.



8. After both pucks have crossed the second dotted line, what is the relationship between the **kinetic energies** K_1 and K_2 for pucks #1 and #2? (**ave = 62%**)
   (a) $K_1 < K_2$
   (b) $\boxed{K_1 = K_2}$ $K = K_i + \Delta K$ where $K_{1i} = K_{2i}$ and $\Delta K_1 = \Delta K_2 = Fd$
   (c) $K_1 > K_2$
   (d) Insufficient information

9. After both pucks have crossed the second dotted line, what is the relationship between the **magnitudes** of the **impulses** I_1 and I_2 exerted on pucks #1 and #2 between the dotted lines? (**ave = 50%**)
   (a) $\boxed{I_1 < I_2}$ $\vec{I} = \vec{F}\Delta t$ where $\Delta t_1 < \Delta t_2$ because puck #1 reaches 2nd dotted line first
   (b) $I_1 = I_2$
   (c) $I_1 > I_2$
   (d) Insufficient information

10. A rotating disk is slowing down, where **point 1** is **half-way** to the rim and **point 2** is on the **rim**. Which statement is true? (**ave = 67%**)
    (a) Point 1 has the same angular acceleration α as point 2.
    (b) Point 1 has smaller angular acceleration α than point 2.
    (c) Point 1 has the same parallel acceleration $a_{//}$ as point 2. (Remember $a_{//} = r\alpha$)
    (d) Point 1 has a larger parallel acceleration $a_{//}$ than point 2.
    (e) None of the above.

11. If $\vec{r} = \left(2\hat{i} - 3\hat{j}\right)$ m and $\vec{F} = \left(-2\hat{i} + \hat{j}\right)$ N then the torque $\vec{\tau}$ is given by: (**ave = 66%**)
    (a) $-7\hat{k}$ Nm
    (b) $8\hat{k}$ Nm
    (c) $\boxed{-4\hat{k}\text{ Nm}}$ $\quad \vec{r} \times \vec{F} = \left(2\hat{i} - 3\hat{j}\right) \times \left(-2\hat{i} + \hat{j}\right) = 2\hat{i} \times \hat{j} + 3\hat{j} \times 2\hat{i} = 2\hat{k} - 6\hat{k} = -4\hat{k}$
    (d) $-8\hat{k}$ Nm
    (e) None of the above.

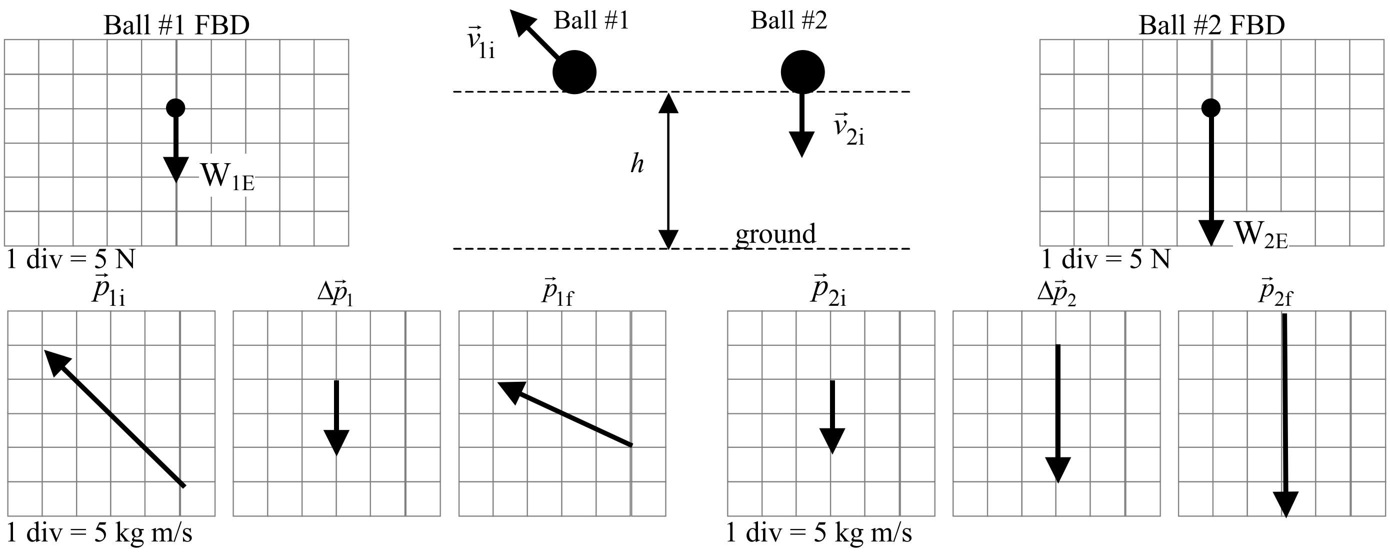
12. A **ring** rolls along the floor without slipping. What is the **relationship** between its **rotational** kinetic energy and its **translational** kinetic energy? (**ave = 57%**)
    (a) $\boxed{K_{\text{rot}} = K_{\text{trans}}}$ $\quad \dfrac{K_{\text{rot}}}{K_{\text{trans}}} = \dfrac{\frac{1}{2}\beta m v^2}{\frac{1}{2} m v^2} = \beta = 1$
    (b) $K_{\text{rot}} = 2K_{\text{trans}}$
    (c) $K_{\text{rot}} = \frac{1}{2}K_{\text{trans}}$
    (d) $K_{\text{rot}} = 1.5K_{\text{trans}}$
    (e) None of the above.

13. A student rides on the outside of a rotating merry-go-round and holds a ball in her hand. If she **throws** the ball in the direction of her motion, the **angular velocity** of the **merry-go-round** will (**ave = 73%**)
    (a) Increase
    (b) **Decrease** If L_{ball} increases ("thrown" increases v), then L_{merry} decreases.
    (c) Remain the same
    (d) Insufficient information

14. A **<u>DISK</u>** rotates with initial angular velocity ω_0. A **<u>RING</u>** of **equal mass** rotates in the opposite direction with $-\omega_0$. If the disk and ring "collide" and eventually rotate together, what is the **final** ω_f? (**ave = 23%**)
    (a) $-\frac{3}{2}\omega_0$
    (b) $-\frac{1}{2}\omega_0$
    (c) zero
    (d) $\frac{1}{2}\omega_0$
    (e) None of the above. $\quad L_i = L_f \Rightarrow \frac{1}{2}mR^2\left(\omega_0\right) - mR^2\left(\omega_0\right) = \left(\frac{1}{2}mR^2 + mR^2\right)\left(\omega_f\right) \quad \therefore \boxed{\omega_f} = -\frac{1}{3}\omega_0$

15. Wheel #1 (**radius *R***, mass *m*) and wheel #2 (**radius 2*R***, mass *m*) can each freely rotate when a force is applied tangential to the outside rim. What is the relationship between the **angular acceleration** α_1 of wheel #1 and α_2 of wheel #2 for the **same applied force F**? (**ave = 34%**)
    (a) $\boxed{\alpha_2 = \frac{1}{2}\alpha_1}$ $\quad \tau = I\alpha \Rightarrow rF = \left(mr^2\right)\alpha \quad \boxed{\alpha} = \dfrac{F}{mr}$ and $\boxed{\dfrac{\alpha_2}{\alpha_1}} = \dfrac{r_1}{r_2} = \dfrac{1}{2}$
    (b) $\alpha_2 = \alpha_1$
    (c) $\alpha_2 = 2\alpha_1$
    (d) $\alpha_2 = 4\alpha_1$
    (e) None of the above.

16. At time t = 0 s, two balls are dropped from a height h above the ground. Ball #1 (**1 kg**) has an **initial velocity** $\vec{v}_{1i} = (-20\hat{i} + 20\hat{j})$ m/s and ball #2 (**2 kg**) has an **initial velocity** $\vec{v}_{2i} = (-5\hat{j})$ m/s . **(25 pts) (ave = 89%)**



a. Draw the **free body diagrams** for the balls using the two subscript notation.

b. Draw the **initial momenta** $\vec{p}_i$ for each ball on the grids.

c. Draw the **change in momenta** $\Delta\vec{p}$ during **1 s of travel** for each ball on the grids.
Remember that $\Delta\vec{p} = \vec{I} = \vec{F}_{net}\Delta t$.

d. Draw the **final momenta** $\vec{p}_f$ after **1 s** of travel for each ball on the grids.

e. Find the **initial kinetic energies** K_i of each ball.

$$\boxed{K_{1i}} = \tfrac{1}{2}m_1(v_{1x,i}^2 + v_{1y,i}^2) = \tfrac{1}{2}(1\text{ kg})\left[(-20\text{ kg m/s})^2 + (20\text{ kg m/s})^2\right] = \boxed{400\text{ J}}$$

$$\boxed{K_{2i}} = \tfrac{1}{2}m_2(v_{2x,i}^2 + v_{2y,i}^2) = \tfrac{1}{2}(2\text{ kg})\left[(5\text{ kg m/s})^2\right] = \boxed{25\text{ J}}$$

f. Find the **final kinetic energies** K_f of each ball after 1 s of travel.

$$\boxed{K_{1f}} = \frac{p_{1f}^{\,2}}{2m_1} = \frac{p_{1x,f}^2 + p_{1y,f}^2}{2m_1} = \frac{(-20\text{ kg m/s})^2 + (10\text{ kg m/s})^2}{2(1\text{ kg})} = \boxed{250\text{ J}}$$

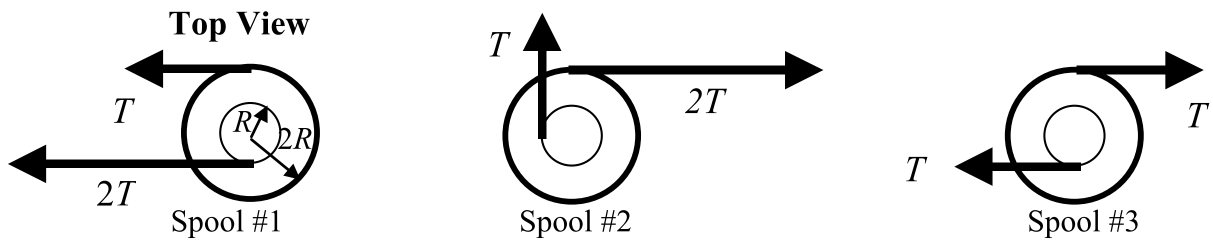
$$\boxed{K_{2f}} = \frac{p_{2f}^{\,2}}{2m_2} = \frac{p_{2x,f}^2 + p_{2y,f}^2}{2m_2} = \frac{(-30\text{ kg m/s})^2}{2(2\text{ kg})} = \boxed{225\text{ J}}$$

g. Find the **changes in kinetic energy** ΔK of each ball after 1 s of travel.

$$\boxed{\Delta K_1} = K_{1,f} - K_{1,i} = 250\text{ J} - 400\text{ J} = \boxed{-150\text{ J}}$$

$$\boxed{\Delta K_2} = K_{2,f} - K_{2,i} = 225\text{ J} - 25\text{ J} = \boxed{200\text{ J}}$$

17. Three identical spools are resting on a horizontal, frictionless table as shown in top view. They are each being pulled by the tension forces as shown at varying locations on the spool. (Imagine that a string is being pulled that is wound around the spool.) **(15 pts)** (**ave = 82%**)



a. Find the **values** of the **NET torques** for each spool. Assume that a **positive** torque points **into the page**.

$\boxed{\tau_1} = \vec{R}\times 2\vec{T} - 2\vec{R}\times\vec{T} = \boxed{0}$

$\boxed{\tau_2} = \vec{R}\times\vec{T} + 2\vec{R}\times 2\vec{T} = \boxed{5RT}$ into page

$\boxed{\tau_3} = \vec{R}\times\vec{T} + 2\vec{R}\times\vec{T} = \boxed{3RT}$ into page

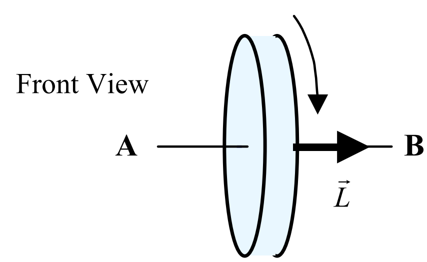
b. Give the ranking of the <u>**magnitudes**</u> of the **NET angular accelerations** $\alpha_1, \alpha_2, \alpha_3$ of the spools. Remember that $\vec{\tau} = I\vec{\alpha}$.

$\boxed{\alpha_2 > \alpha_3 > \alpha_1}$ because $\alpha \propto \tau$

c. Give the ranking of the <u>**magnitudes**</u> of the **NET linear accelerations** a_1, a_2, a_3 of the spools.

$\boxed{a_1 > a_2 > a_3}$ $a \propto F$ and $F_1(=3T) > F_2\left(=\sqrt{5}T\right) > F_3(=0)$

18. If a wheel rotates as shown, <u>**DRAW**</u> & label the angular momentum vector $\vec{L}$ on the picture with the wheel. **(20 pts)** (**ave = 79%**)



<u>**PART I**</u>: If you pull **OUT** of the page **at A** and push **IN at B**, <u>**draw**</u> & label the <u>**THREE**</u> $\vec{\tau} = \vec{r}\times\vec{F}$ vectors for <u>**points A & B**</u> in the top boxes below the drawing.

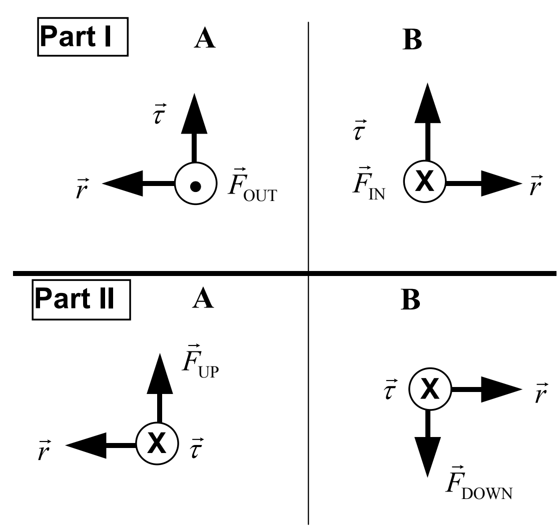
Direction $\vec{\tau}$ = UP

Due to this torque the wheel's axis moves <u>**CCW**</u> from the front view.

<u>**PART II**</u>: If you push **UP at A** and pull **DOWN at B**, <u>**draw**</u> & label the <u>**THREE**</u> $\vec{\tau} = \vec{r}\times\vec{F}$ vectors for <u>**points A & B**</u> in the bottom boxes below the drawing.

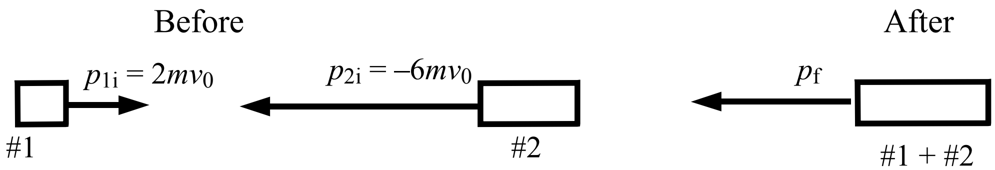
Direction $\vec{\tau}$ = INTO page

Due to this torque the wheel's axis moves <u>**CCW**</u> from the TOP view.



19. Block #1 (mass $2M$, initial velocity $+v_0$) travels **right** and block #2 (mass $3M$, initial velocity $-2v_0$) travels **left**. The blocks are on a frictionless surface and have a **perfectly inelastic** collision. **(25 pts)** **(ave = 77%)**

(a) Draw **two pictures** of the blocks for **before** and **after** the collision. Label the blocks #1 and #2 and draw and label the relevant **momentum** vectors (p_{1i}, p_{2i}, p_f). Scale the **lengths** of the momentum vectors appropriately.



(b) Using conservation of momentum, find the **final velocity** v_f of the combined blocks immediately after the collision in terms of the given variables (M, v_0).

For $\Delta p = 0 \;\Rightarrow\; p_{1i} + p_{2i} = p_{1f} + p_{2f}$

$$2Mv_0 + (3M)(-2v_0) = (2M + 3M)v_f$$

$$-4Mv_0 = 5Mv_f$$

$$\boxed{v_f = -\frac{4v_0}{5}}$$

(c) Find the **impulse I₁** exerted on Block #1 due to the collision in terms of the given variables (M, v_0).

$$\boxed{I_1} = \Delta p_1 = p_{1f} - p_{1i} = \boxed{m_1\left(v_{1f} - v_{1i}\right)}$$

$$I_1 = 2M\left(\left(-\frac{4v_0}{5}\right) - v_0\right) \quad \text{using part (b)}$$

$$\boxed{I_1 = -\frac{18Mv_0}{5}}$$

(d) After the collision, the blocks then compress a **spring** with spring constant k and momentarily stop. Using the Work-Energy Theorem #2, FIRST find the **GENERAL FORMULA** for the **compression distance** $\boldsymbol{x}$ for a block of mass m and velocity v.

$$\Delta K + \Delta U + f\Delta s = W_{ext} \qquad \text{where } W_{ext} = f = 0$$

$$\left(K_f - K_i\right) + \left(U_{Sf} - U_{Si}\right) = 0 \qquad \text{where } K_i \text{ equals the energy after the collision}$$

$$\left(0 - \tfrac{1}{2}mv^2\right) + \left(\tfrac{1}{2}kx^2 - 0\right) = 0$$

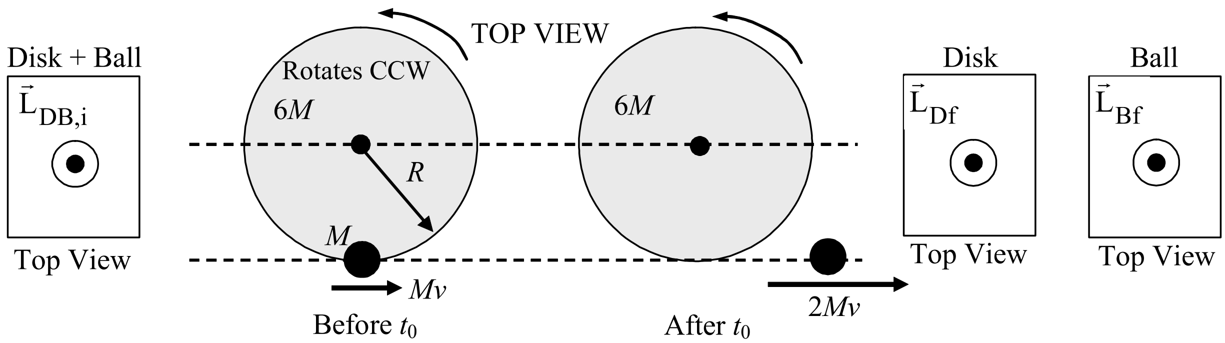
$$\boxed{x = v\sqrt{\frac{m}{k}}}$$

(e) Now, find the **compression distance** $\boldsymbol{x}$ for the two blocks in terms of any given variables (M, v_0, k).

$$\boxed{x} = v\sqrt{\frac{m}{k}} = \frac{4v_0}{5}\sqrt{\frac{5M}{k}} = \boxed{4v_0\sqrt{\frac{M}{5k}}}$$

20. A **disk** of **mass 6*M*** and radius R is **horizontally** mounted with a pivot at the center. A **ball** of **mass *M*** is attached at radius R to the disk. The disk and ball rotate together with **initial angular velocity** ω_0 in a counterclockwise direction. At time t_0, the ball is **shot away** from the disk with **twice** its initial velocity, and the disk continues to rotate in the same direction with **final angular velocity** ω_f. **(25 pts) (ave = 77%)**

(a) Diagrams of the disk and ball both **before** and **after** the ball's separation are shown below. Draw the **angular momentum** vector directions before and after the ball leaves the disk as indicated by the boxes.



(b) Find the initial **moments of inertia** I_D of the **disk,** I_B of the **ball,**and I_{DB} of the **disk+ball** system in terms of ***M* and *R*.**

$$\boxed{I_D} = \tfrac{1}{2}mr^2 = \tfrac{1}{2}(6M)R^2 = \boxed{3MR^2}$$

$$\boxed{I_B} = mr^2 = \boxed{MR^2} \quad \text{(same as point source or ring)}$$

$$\boxed{I_{DB}} = I_D + I_B = \boxed{4MR^2}$$

(c) Find the **initial angular momentum** L_i of the disk + ball system in terms of ***M*, *R*, and** ω_0. Assume that **positive L** values point **out** of the page (counterclockwise rotation).

$$\boxed{L_i} = I_{DB}\omega_0 = \boxed{4MR^2\omega_0}$$

(d) Find the **final angular momenta** L_{Df} of the **disk** and L_{Bf} of the **ball** in terms of ***M*, *R*,** ω_0**, and** ω_f.

$$\boxed{L_{Df}} = I_D\omega_f = \boxed{3MR^2\omega_f}$$

$$L_{Bf} = \vec{r} \times \vec{p}_{Bf} = R(Mv_f) \quad \text{where } v_f = 2\omega_0 R \quad \text{(twice initial speed)}$$

$$\boxed{L_{Bf} = 2MR^2\omega_0}$$

(e) Using conservation of angular momentum, find the **final angular velocity** ω_f of the disk in terms of any given variables (M, R, ω_0).

$$\vec{L}_i = \vec{L}_f$$

$$4MR^2\omega_0 = 3MR^2\omega_f + 2MR^2\omega_0$$

$$\boxed{\omega_f = \tfrac{2}{3}\omega_0}$$