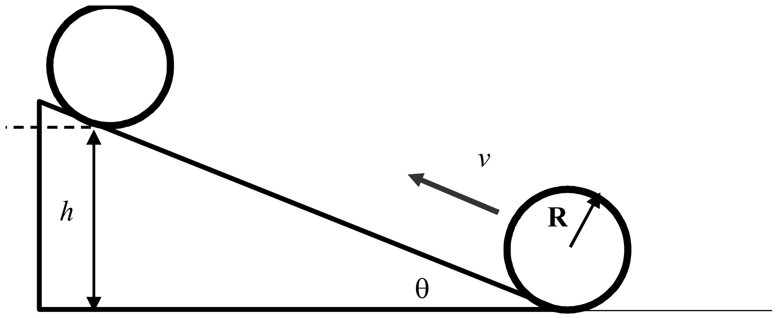
HOMEWORK Solutions #8: Rotation II (Phys 207, Fall 2005)

QUIZ #8 on HW#8 this Thursday (11/17) at BEGINNING of class

Problem #1: Convert Kinetic to Potential Energy

A **RING** rolls on a horizontal surface and then rolls up an incline without slipping. Assume that the ring has **mass *m***, **radius *R***, and initial **velocity *v***.

(a) Draw **pictures** of the ring both **before** and **after** it rolls up the incline. Draw and label any relevant variables (h, R, v).



(b) Find the total **initial kinetic *K*i** energy of the ring.

$$K_i = K_{\text{i,trans}} + K_{\text{i,rot}} = \tfrac{1}{2} mv^2 (1+\beta) \quad \text{where } \beta = 1 \text{ for ring}$$

$$\boxed{K_i} = \tfrac{1}{2} mv^2 (1+1) = \boxed{mv^2}$$

(c) Using the Work-Energy Theorem #2, find the **height *h*** of the ring on the incline when it is moving with **one third** its initial velocity.

$$\Delta K + \Delta U + f\Delta s = W_{\text{ext}} \quad \text{where } f = W_{\text{ext}} = 0$$

$$(K_{\text{f}} - K_{\text{i}}) + (U_{\text{f}} - U_{\text{i}}) = 0$$

$$\left[m\left(\tfrac{v}{3}\right)^2 - mv^2 \right] + (mgh - 0) = 0$$

$$mgh = mv^2 \left(1 - \tfrac{1}{9}\right)$$

$$\boxed{h = \frac{8v^2}{9g}}$$

(d) If the rolling ring were replaced by a **disk** with the same initial velocity, would this height be greater, smaller, or the same? Explain why.

The height for the **disk** would be **lower** because it has a **lower** β and, hence, lower rotational kinetic energy ($K_{rot} = \beta mv^2/2$, where β = 1/2 for disk vs. β = 1 for ring).

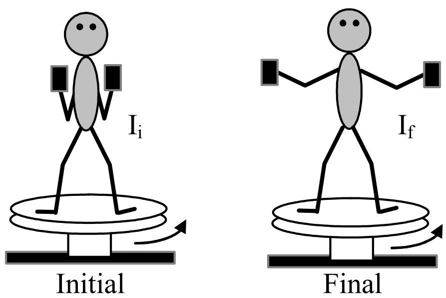
<u>Problem #2</u>: Changing Moment of Inertia *I*

A girls stands on a frictionless, rotating platform and holds weights **close** to her body. The platform rotates with initial angular velocity ω_i and the system's initial moment of inertia is I_i. She then **extends** her arms as shown in the picture below. The platform rotates **counterclockwise** from the **top view**.

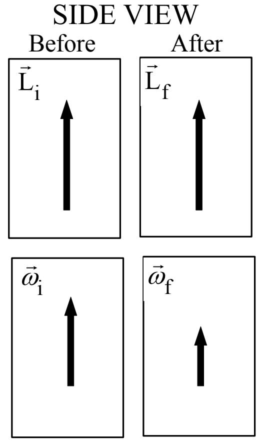
(a) Draw the **vector directions** in the side-view boxes for the system's initial and final angular **velocities** (ω_i , ω_f) AND angular **momenta** (L_i , L_f). Check that the relative lengths of the vectors are consistent.

<u>Note</u>: $L_i = L_f$ since $\tau_{ext} = 0$

SIDE VIEW



SIDE VIEW



<u>Note</u>: The vector lengths should be such that $\omega_i > \omega_f$ and $L_i = L_f$.

(b) The **moment of inertia** of the girl ____**<u>Increases</u>**______ (increases or decreases) when she extends her arms.

Remember that the moment of inertia increases when the radial distance of mass from the rotation axis increases, i.e., $I = mr^2$. When the girl's arms are extended, the **amount of mass at larger radial distance increases** and, therefore, her moment of inertia must also increase.

(c) The **angular velocity** of the girl _____**<u>Decreases</u>** ________ (increases or decreases) when she extends her arms.

Remember that **conservation of angular momentum** states that the product of the moment of inertia and angular velocity is constant, i.e., $L = I_i\omega_i = I_f\omega_f$. When the girl extends her arms and the moment of inertia increases, then her angular velocity must decrease to conserve angular momentum.

(d) Find an algebraic expression for the **change in kinetic energy ΔK** of the girl + platform system after the weights are extended outward (in terms of ω_i , ω_f, I_i , I_f).

$$\boxed{\Delta K} = K_f - K_i = \boxed{\tfrac{1}{2} I_f \omega_f^2 - \tfrac{1}{2} I_i \omega_i^2}$$

(e) If the angular speed of the system is **halved** when the girl extends her arms, find **ΔK** in terms of $\boldsymbol{\omega_i}$ **and** $\boldsymbol{I_i}$.

$$\Delta K = \tfrac{1}{2} I_f \omega_f^2 - \tfrac{1}{2} I_i \omega_i^2$$

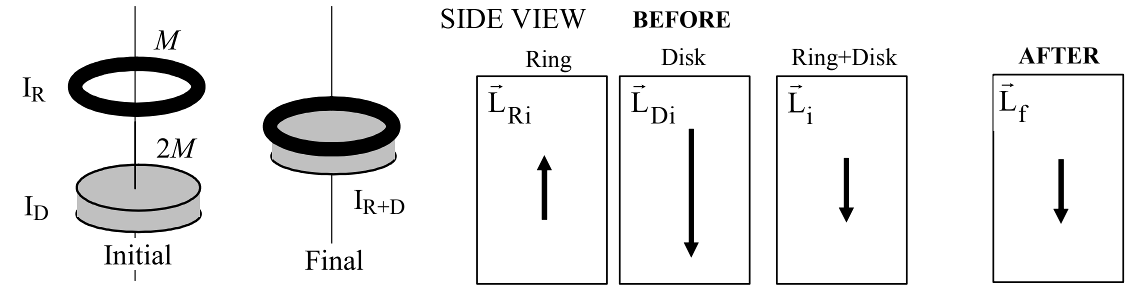
$$\Delta K = \tfrac{1}{2}(2I_i)\left(\frac{\omega_i}{2}\right)^2 - \tfrac{1}{2} I_i \omega_i^2 \qquad \text{where } I_f = 2I_i \text{ due to } \vec{L} \text{ conservation: } I_f\omega_f = (2I_i)\left(\frac{\omega_i}{2}\right) = I_i\omega_i$$

$$\boxed{\Delta K} = \tfrac{1}{2} I_i \omega_i^2 \left[\tfrac{1}{2} - 1\right] = \boxed{-\tfrac{1}{4} I_i \omega_i^2}$$

<u>Problem #3</u>: Perfectly Inelastic "Collision" of Ring and Disk

A **ring** (mass M) and **disk** (mass $2M$) with equal radii R are spinning on frictionless bearings in **<u>opposite</u>** directions. The **ring** is rotating counterclockwise with $+\omega$ and the **disk** clockwise with $\mathbf{-2\omega}$. They are then brought together and due to friction spin at the same final angular velocity ω_f.

(a) Diagrams of the disk and ring both **before** and **after** the "collision" are shown below. After completing parts (c) and (d) below, draw the indicated **angular momentum** vectors before and after the collision in the boxes. **Check** that the relative **<u>lengths</u>** of the vectors are consistent.



(b) Find the **moments of inertia** I_R of the **ring,** I_D of the **disk,** and I_{RD} of the **ring+disk** system in terms of ***M* and *R*.**

$$\boxed{I_R} = mr^2 = \boxed{MR^2}$$

$$\boxed{I_D} = \tfrac{1}{2}mr^2 = \tfrac{1}{2}(2M)R^2 = \boxed{MR^2}$$

$$\boxed{I_{RD}} = I_R + I_D = \boxed{2MR^2}$$

(c) Find the **<u>initial</u> angular momenta** L_{Ri} of the **ring,** L_{Di} of the **disk,** and total Li of the **ring+disk** system in terms of ***M*, *R*, and ω**. Assume that **positive L** values point **upward** (counterclockwise rotation).

$$\boxed{L_{Ri}} = I_R\omega_{Ri} = \boxed{MR^2\omega}$$

$$\boxed{L_{Di}} = I_D\omega_{Di} = MR^2(-2\omega) = \boxed{-2MR^2\omega}$$

$$\boxed{L_i} = L_{Ri} + L_{Di} = MR^2\omega - 2MR^2\omega = \boxed{-MR^2\omega}$$

(d) Find the **<u>final</u> angular momentum** L_f of the ring + disk system in terms of ***M*, *R*, and ω_f**.

$$\boxed{L_f} = I_{RD}\omega_f = \boxed{2MR^2\omega_f}$$

(e) Using conservation of angular momentum, find the **<u>final</u> angular velocity ω_f** of the ring + disk system in terms of any given variables (M, R, ω). State the **direction** of ω_f.

$$\vec{L}_i = \vec{L}_f$$

$$-MR^2\omega = 2MR^2\omega_f$$

$$\boxed{\omega_f = -\tfrac{1}{2}\omega}$$ points downward

(f) In the above example, the ring and disk have a non-zero final ω_f. Find the **<u>ratio</u>** of the initial angular velocities of the **ring to the disk** ($\omega_{Ri} / \omega_{Di}$) so that they **<u>stop</u> rotating** when brought together, i.e. $L_f = 0$.

The initial angular momentum of the system $\mathbf{L_i}$ **must equal zero** in order for the final angular momentum L_f, and therefore angular velocity ω_f, to equal zero.

$$L_i = I_R\omega_{Ri} + I_D\omega_{Di} = 0$$

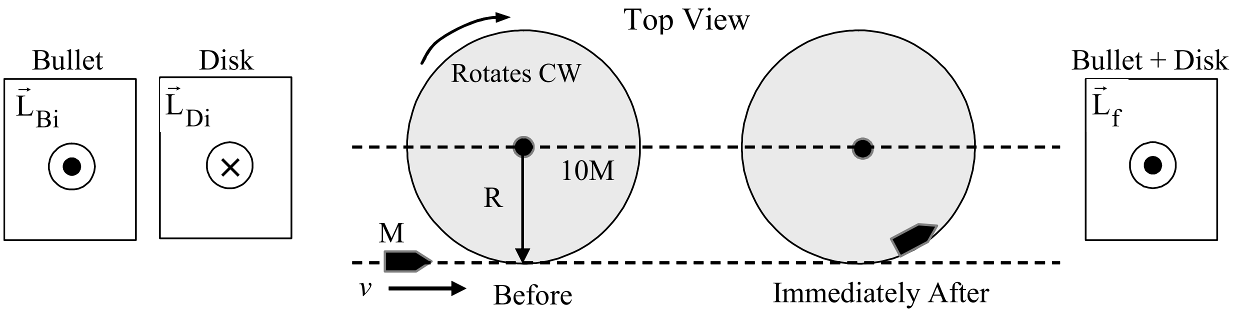
$$I_R\omega_{Ri} = -I_D\omega_{Di}$$

$$\boxed{\frac{\omega_{Ri}}{\omega_{Di}}} = -\frac{I_D}{I_R} = -\left(\frac{MR^2}{MR^2}\right) = \boxed{-1}$$ rotate opposite directions

Problem #4: Perfectly Inelastic "Collision" of Bullet with Disk

A **disk** of mass **10M** and radius **R** is horizontally mounted with a pivot at the center, and it rotates clockwise with $-\omega$. A **bullet** of mass **M** and initial **speed** v is shot horizontally such that it collides with the disk at its radius R and then remains lodged.

(a) Top-view diagrams of the disk and bullet both **before** and **after** the collision are shown below. **Assume** that the disk reverses direction after the collision. Draw the **angular momentum** vector directions of the bullet and disk before the collision and of the bullet+disk system after the collision in the boxes.



(b) Find the **initial angular momentum** L_{Di} of the **disk** in terms of ***M*, *R*, and** ω. Assume that **positive L** values point **out of the page** (counterclockwise rotation).

$L_{Di} = I_D \omega_i$ where $I_D = \frac{1}{2}mr^2 = \frac{1}{2}(10M)R^2$ and $\omega_i = -\omega$

$$\boxed{L_{Di} = -5MR^2\omega}$$

(c) Find the **initial angular momentum** L_{Bi} of the **bullet** in terms of ***M*, *R*, and *v***.

$L_{Bi} = |\vec{r} \times \vec{p}_{Bi}|$

$$\boxed{L_{Bi} = R(Mv)}$$

(d) Find the **final angular momentum** L_f of the bullet + disk system in terms of ***M*, *R*, and** ω_f.

$L_f = (I_D + I_B)\omega_f = (5MR^2 + MR^2)\omega_f$ where $I_B = I_{point} = MR^2$

$$\boxed{L_f = 6MR^2\omega_f}$$

(e) Using conservation of angular momentum, find the **final angular velocity** ω_f of the bullet + disk system in terms of any given variables (M, R, ω, v). Simplify the expression and state the **direction** of ω_f.

$\vec{L}_{Di} + \vec{L}_{Bi} = \vec{L}_f$

$-5MR^2\omega + RMv = 6MR^2\omega_f$

$-5R\omega + v = 6R\omega_f$ (divide by MR)

$$\boxed{\omega_f = \frac{v - 5R\omega}{6R}}$$ **Out** of page.