# <u>**HOMEWORK Solutions #6**</u>: Momentum (Phys 207, Spring 2005)

**QUIZ #6 on HW#6** on Thursday (11/3/05) at BEGINNING of class

<u>**Problem #1**</u>**: 1-D Perfect Inelastic Collision (20 pts)**

A semitruck (#1) and car (#2) are driving towards each other as shown below and have a **perfectly inelastic** head-on collision. After the collision, the two vehicles skid a distance Δs on the pavement (coefficient of kinetic friction = $\mu_k$) before coming to a complete stop.


(a) If the initial **kinetic energies** of the two vehicles are <u>**equal**</u>, then find an **algebraic expression** for the **ratio** $v_{1i} / v_{2i}$ of the initial velocities for the semitruck $v_{1i}$ and car $v_{2i}$. Assume that the semitruck has mass $m_1$ and the car has mass $m_2$.

$$K_{1i} = K_{2i} \quad \Rightarrow \quad \tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_2 v_{2i}^2 \quad \Rightarrow \quad \boxed{\frac{v_{1i}}{v_{2i}} = \sqrt{\frac{m_2}{m_1}}}$$

(b) Using conservation of momentum, find an **algebraic expression** for the **velocity** $v_f$ of the combined semitruck/car heap immediately after the collision. (Use $\pm v$ for right/left motion.)

For $\Delta p = 0 \;\Rightarrow\; \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$

$$m_1 v_{1i} - m_2 v_{2i} = (m_1 + m_2) v_f$$

$$\boxed{v_f = \frac{m_1 v_{1i} - m_2 v_{2i}}{m_1 + m_2}}$$

(d) Find the **numerical value** for the **velocity** $v_f$ of the crash heap if the car initially travels at 30 m/s (or ~67 mph), and the truck weighs 4 times the car ($m_1 = 4m_2$). Use parts (a) and (b).

$$v_f = \frac{m_1 v_{1i} - m_2 v_{2i}}{m_1 + m_2} \quad \text{where } m_1 = 4m_2 \quad \text{and} \quad v_{1i} = v_{2i}\sqrt{\frac{m_2}{m_1}} = 30 \text{ m/s}\sqrt{\frac{m_2}{4m_2}} = 15 \text{ m/s}$$

$$\boxed{v_f} = \frac{(4m_2)(15 \text{ m/s}) - m_2 (30 \text{ m/s})}{4m_2 + m_2} = \boxed{6 \text{ m/s}}$$

(e) Using the Work-Energy Theorem #2, find an **algebraic expression** for the **skid distance Δs**. Remember that here the "initial" kinetic energy refers to the energy just after the collision.

$$\Delta K + \Delta U + f\Delta s = W_{ext} \quad \text{where } W_{ext} = \Delta U = 0$$

$$(K_f - K_i) + f\Delta s = 0 \qquad \text{where } f = \mu_k (m_1 + m_2) g$$

$$\left(0 - \tfrac{1}{2}(m_1 + m_2) v_f^2\right) + \mu_k (m_1 + m_2) g \Delta s = 0 \quad \Rightarrow \quad \boxed{\Delta s = \frac{v_f^2}{2\mu_k g}}$$

(f) Find the **numerical value** for the **skid distance Δs** of the crash heap if $\mu_k = 0.5$.

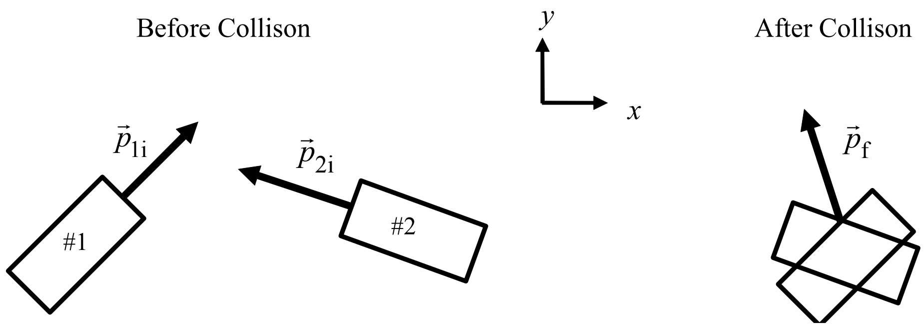
$$\boxed{\Delta s} = \frac{v_f^2}{2\mu_k g} = \frac{(6 \text{ m/s})^2}{2(0.5)(9.8 \text{ m/s}^2)} = \boxed{3.7 \text{ m}}$$

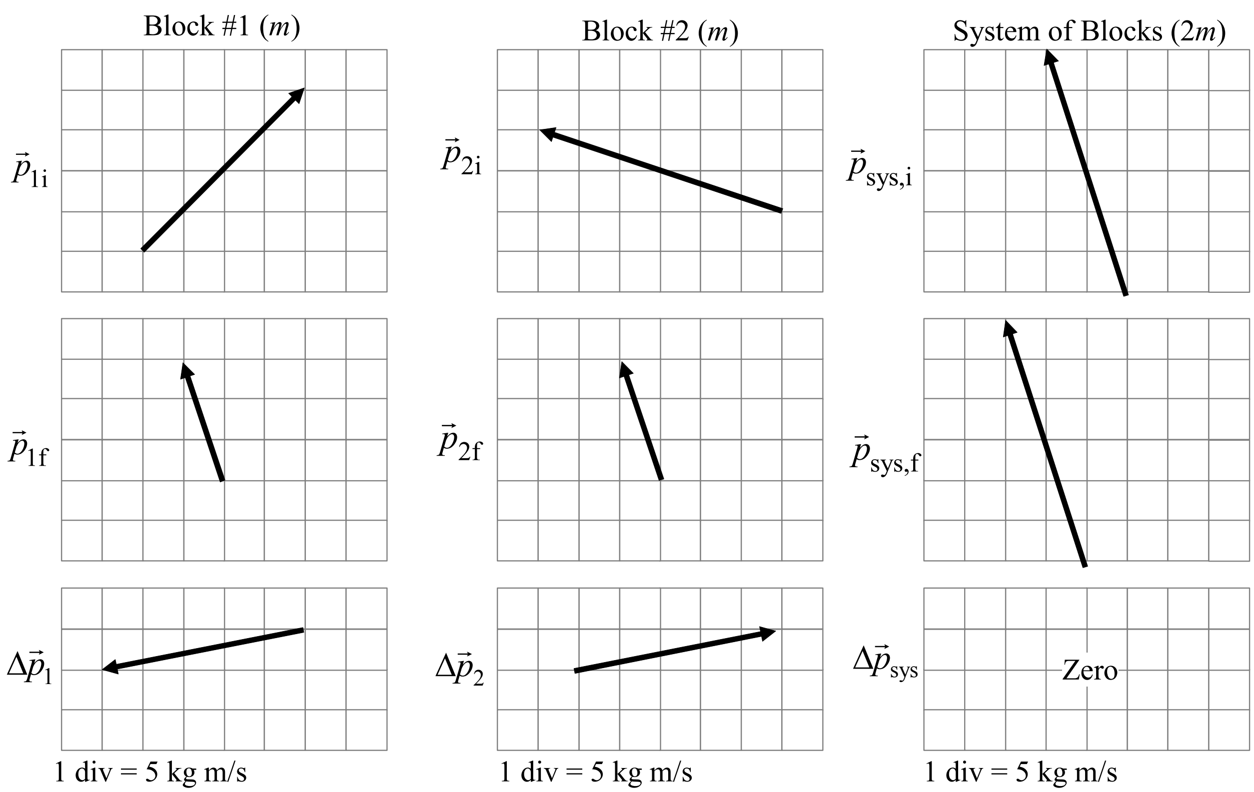
**<u>Problem #2</u>: 2-D Perfect Inelastic Collision (40 pts)**

Two blocks are moving on a frictionless, horizontal table. Block #1 (2 kg) has an **initial velocity** $\vec{v}_{1i} = (10\hat{i} + 10\hat{j})$ m/s and block #2 (2 kg) has an **initial velocity** $\vec{v}_{2i} = (-15\hat{i} + 5\hat{j})$ m/s .

The two blocks then collide and stick together in a perfectly inelastic collision.

a. Draw diagrams of the blocks below for **before** and **after** the collision. Label the blocks and draw the relevant momentum vectors $\vec{p}_{1i}$, $\vec{p}_{2i}$, $\vec{p}_f$ in each picture.



b. Draw the **initial momenta** $\vec{p}_{\rm i}$ for each block and the system on the grids and write their values below.

$$\boxed{\vec{p}_{\rm 1i}} = m\vec{v}_{\rm 1i} = (2\text{ kg})(10\hat{\imath} + 10\hat{\jmath})\text{ m/s} = \boxed{20\hat{\imath} + 20\hat{\jmath}\text{ kg m/s}}$$

$$\boxed{\vec{p}_{\rm 2i}} = m\vec{v}_{\rm 2i} = (2\text{ kg})(-15\hat{\imath} + 5\hat{\jmath})\text{ m/s} = \boxed{-30\hat{\imath} + 10\hat{\jmath}\text{ kg m/s}}$$

$$\boxed{\vec{p}_{\rm sys,i}} = \vec{p}_{\rm 1i} + \vec{p}_{\rm 2i} = \boxed{-10\hat{\imath} + 30\hat{\jmath}\text{ kg m/s}}$$

c. Draw the **final momenta** $\vec{p}_{\rm f}$ for each block and the system on the grids and write their values below.

$\boxed{\vec{p}_{\rm sys,f}} = \vec{p}_{\rm sys,i} = \boxed{-10\hat{\imath} + 30\hat{\jmath}\text{ kg m/s}}$ using conservation of momentum (no external forces)

$\boxed{\vec{p}_{\rm 1f}} = \boxed{\vec{p}_{\rm 2f}} = \frac{1}{2}\vec{p}_{\rm sys,i} = \boxed{-5\hat{\imath} + 15\hat{\jmath}\text{ kg m/s}}$ since blocks have same mass

d. Draw the **change in momenta** $\Delta\vec{p}$ for each block and the system on the grids and write their values below.

$$\boxed{\Delta\vec{p}_1} = \vec{p}_{\rm 1f} - \vec{p}_{\rm 1i} = (-5\hat{\imath} + 15\hat{\jmath}) - (20\hat{\imath} + 20\hat{\jmath})\text{ kg m/s} = \boxed{-25\hat{\imath} - 5\hat{\jmath}\text{ kg m/s}}$$

$$\boxed{\Delta\vec{p}_{\rm 2i}} = \vec{p}_{\rm 2f} - \vec{p}_{\rm 2i} = (-5\hat{\imath} + 15\hat{\jmath}) - (-30\hat{\imath} + 10\hat{\jmath})\text{ kg m/s} = \boxed{25\hat{\imath} + 5\hat{\jmath}\text{ kg m/s}}$$

$\boxed{\Delta\vec{p}_{\rm sys,i}} = 0$ using conservation of momentum (no external forces)

e. What observation can you make about **changes in momenta** $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$? How does this relate to the forces on each block and Newton's 3rd Law pairs?

$\boxed{\Delta\vec{p}_1 = -\Delta\vec{p}_2}$ because $\Delta\vec{p} = \vec{F}\Delta t$ and $\vec{F}_{12} = \vec{F}_{21}$

Blocks exert equal and opposite forces on each other, resulting in equal and opposite impulses or changes in momenta.

f. Find the **initial kinetic energies** $K_{\rm i}$ of both blocks. Remember $K = \frac{m}{2}(v_{\rm x}^2 + v_{\rm y}^2) = \frac{1}{2m}(p_{\rm x}^2 + p_{\rm y}^2)$.

$$\boxed{K_{\rm 1i}} = \frac{p_{\rm 1i}^2}{2m_1} = \frac{(20\text{ kg m/s})^2 + (20\text{ kg m/s})^2}{2(2\text{ kg})} = \boxed{200\text{ J}}$$

$$\boxed{K_{\rm 2i}} = \frac{p_{\rm 2i}^2}{2m_1} = \frac{(30\text{ kg m/s})^2 + (10\text{ kg m/s})^2}{2(2\text{ kg})} = \boxed{250\text{ J}}$$

g. Find the **final kinetic energies** $K_{\rm f}$ of both blocks.

$\boxed{K_{\rm 1f}} = \boxed{K_{\rm 2f}} = \frac{p_{\rm 1f}^2}{2m_1} = \frac{(5\text{ kg m/s})^2 + (15\text{ kg m/s})^2}{2(2\text{ kg})} = \boxed{62.5\text{ J}}$ since blocks have same mass and velocity

h. Find the **change in kinetic energy** of the system of blocks.

$$\boxed{\Delta K_{\rm sys}} = K_{\rm sys,f} - K_{\rm sys,i} = 2(62.5\text{ J}) - (200\text{ J} + 250\text{ J}) = \boxed{-325\text{ J}}$$

**Problem #3: Elastic Collision with One Mass at Rest (20 pts)**

Block #1 ($m_1$, $v_{1i}$) travels to the right on a frictionless, horizontal table and makes a head-on **elastic** collision with a stationary Block #2 ($m_2$).

(a) Draw below two diagrams of the blocks for **before** and **after** the collision. Label the blocks and draw the relevant momentum vectors ($p_{1i}$, $p_{1f}$, $p_{2f}$) in each picture. In this case, assume that Block #1 has a **larger mass** than Block #2.


(b) If $m_1 = 4m_2$ and block #1 travels at initial speed *v*, then find the **final velocities** of both blocks in terms of *v*. Indicate whether each block travels to the right or left. See class notes for final velocity formulas.

$$\boxed{v_{1f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{4m_2 - m_2}{4m_2 + m_2}\right) v = \boxed{0.6v} \text{ to right}$$

$$\boxed{v_{2f}} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left(\frac{2(4m_2)}{4m_2 + m_2}\right) v = \boxed{1.6v} \text{ to right}$$

(c) Find the **initial and final momenta** of the system for part (c) in terms of $m_2$ and *v*. Check that momentum is conserved.

$$\boxed{p_i} = m_1 v_{1i} = \boxed{4m_2 v}$$

$$\boxed{p_f} = m_1 v_{1f} + m_2 v_{2f} = (4m_2)(0.6v) + m_2(1.6v) = \boxed{4m_2 v}$$

(d) If $m_1 = \frac{1}{4} m_2$ and block #1 travels at initial speed *v*, then find the **final velocities** of both blocks in terms of *v*. Indicate whether each block travels to the right or left.

$$\boxed{v_{1f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{\frac{1}{4} m_2 - m_2}{\frac{1}{4} m_2 + m_2}\right) v = \boxed{-0.6v} \text{ to left}$$

$$\boxed{v_{2f}} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left(\frac{2\left(\frac{1}{4} m_2\right)}{\frac{1}{4} m_2 + m_2}\right) v = \boxed{0.4v} \text{ to right}$$

(e) Find the **initial and final momenta** of the system for part (d) in terms of $m_2$ and *v*. Check that momentum is conserved.

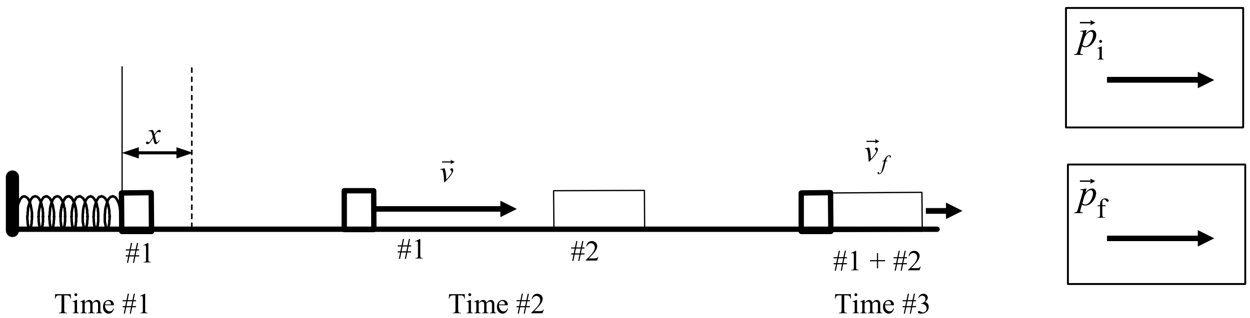
$$\boxed{p_i} = m_1 v_{1i} = \boxed{0.25 m_2 v}$$

$$\boxed{p_f} = m_1 v_{1f} + m_2 v_{2f} = \left(\frac{m_2}{4}\right)(-0.6v) + m_2(0.4v) = \boxed{0.25 m_2 v}$$

**<u>Problem #4</u>: Combination Momentum/Energy Problem (20 pts)**

Block #1 (mass $m$) is pushed against a horizontal spring such that the spring is **compressed** by a **distance** $\boldsymbol{x}$ from its equilibrium length. The block is then released from the compressed spring and slides on a frictionless surface with **velocity** $\boldsymbol{v}$ until it collides with a stationary block #2 (mass $3m$). The collision is perfectly **inelastic**. The two blocks then travel together with **final velocity** $\boldsymbol{v_f}$.

(a) A picture of the problem is shown below for the time periods 1) before Block #1 is released, 2) after Block #1 is released, and 3) after Block #1 collides with Block #2. Draw and label the **velocity** vectors $v$ and $v_f$ on the blocks where necessary. Also, draw the **momentum** vectors of the system of blocks both **before** and **after** the collision in the provided boxes. Pay attention to the relative lengths of vectors.




(b) Using the work-energy theorem #2 (i.e., energy conservation), find an **algebraic** expression for the **velocity** $\boldsymbol{v}$ of block #1 before it collides with block #2.

$\Delta K + \Delta U + f\Delta s = W_{ext}$ where $f\Delta s = W_{ext} = 0$

$(K_f - K_i) + (U_{Sf} - U_{Si}) = 0$

$\left(\frac{1}{2}mv^2 - 0\right) + \left(0 - \frac{1}{2}kx^2\right) = 0$

$$\boxed{v = x\sqrt{\frac{k}{m}}}$$

(c) Using conservation of momentum, find an **algebraic** expression for the **final velocity** $v_f$ of the blocks after the collision. Express it in terms of the initial variables $m$, $k$, $x$.

$p_{1i} + p_{2i} = p_{1f} + p_{2f}$

$mv + 0 = (m + 3m)v_f$

$v_f = \left(\frac{m}{m+3m}\right)v = \left(\frac{1}{4}\right)v$ where $v = x\sqrt{\frac{k}{m}}$ from part (b)

$$\boxed{v_f = \frac{x}{4}\sqrt{\frac{k}{m}}}$$

(d) If the compression **length** $\boldsymbol{x}$ of the spring is **halved**, how does the **final velocity** $v_f$ of the two blocks change?

The final velocity $v_f$ will also **<u>halved</u>** because it is proportional to $x$ in part (c).