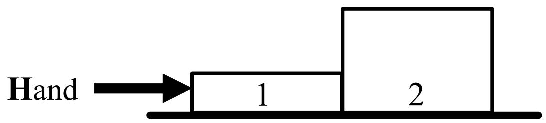
<u>HOMEWORK Solutions #5</u>: Work and Energy (Phys 207, Fall 2005)

QUIZ #5 on HW#5 this Thursday (10/27/05) at BEGINNING of class

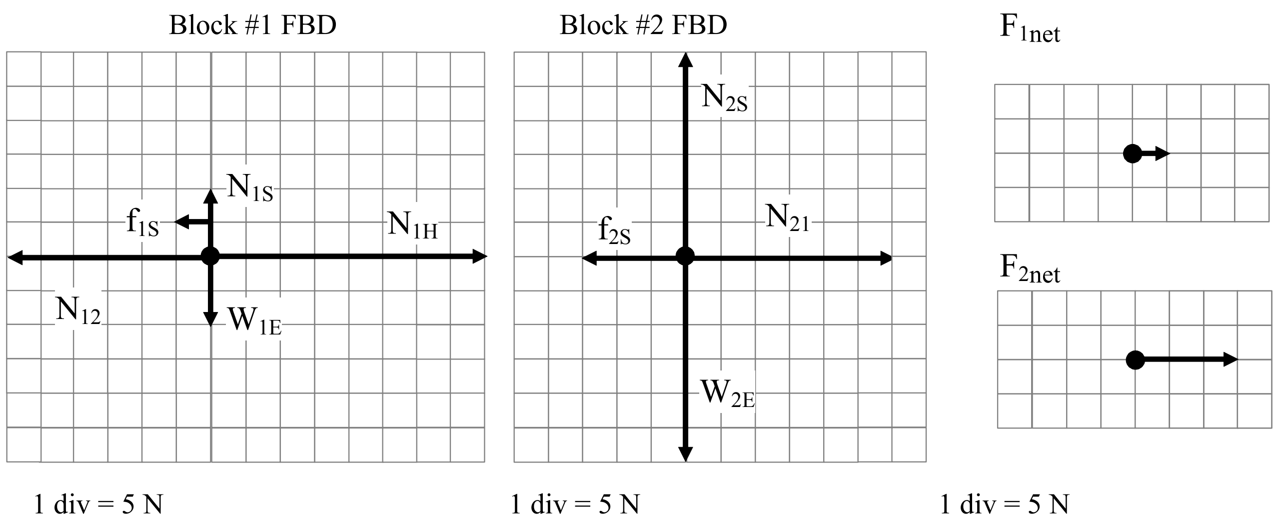
<u>Problem #1</u>: Convert external work to Kinetic Energy

Block #1 (1 kg) and block #2 (3 kg) are being accelerated to the right at **5 m/s²** by a hand pushing on block #1. The **initial velocity** of the two blocks is **4 m/s**. The coefficient of kinetic friction between the blocks and the surface is μ_k **= 0.5.**

(a) Draw the **free body diagrams** for both blocks using the two subscript notation, and draw the **net force** on each block. (assume g = 10 m/s²)



Surface with $\mu_k = 0.5$



(b) Find the **initial** kinetic energies $\boldsymbol{K_{1i}}$ and $\boldsymbol{K_{2i}}$ for blocks #1 and #2, respectively.

$$\boxed{K_{1i}} = \tfrac{1}{2}m_1v^2 = \tfrac{1}{2}(1\text{ kg})(4\text{ m/s})^2 = \boxed{8\text{ J}}$$

$$\boxed{K_{2i}} = \tfrac{1}{2}m_2v^2 = \tfrac{1}{2}(3\text{ kg})(4\text{ m/s})^2 = \boxed{24\text{ J}}$$

(c) If the blocks travel a **distance of 2 m**, find the **change** in kinetic energies ΔK_1 and ΔK_2 for blocks #1 and #2, respectively. <u>Hint</u>: Use the Work-Kinetic Energy theorem.

$$\boxed{\Delta K_1} = W_{1\text{net}} = F_{1\text{net}}\Delta x = (5\text{ N})(2\text{ m}) = \boxed{10\text{ J}}$$

$$\boxed{\Delta K_2} = W_{2\text{net}} = F_{2\text{net}}\Delta x = (15\text{ N})(2\text{ m}) = \boxed{30\text{ J}}$$

(d) Find the **final** kinetic energies $\boldsymbol{K_{1f}}$ and $\boldsymbol{K_{2f}}$ for blocks #1 and #2, respectively. Use your work from parts (b) and (c).

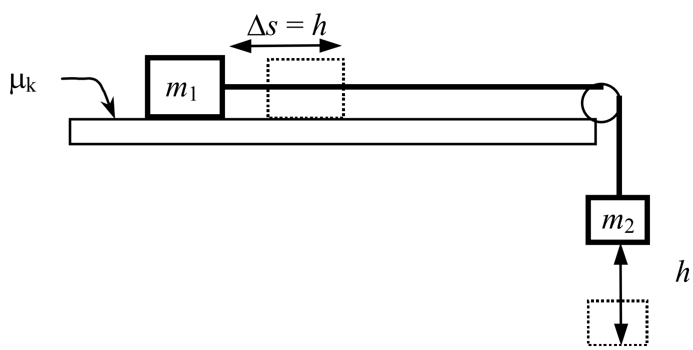
$$\boxed{K_{1f}} = K_{1i} + \Delta K_1 = 8\text{ J} + 10\text{ J} = \boxed{18\text{ J}}$$

$$\boxed{K_{2f}} = K_{2i} + \Delta K_2 = 24\text{ J} + 30\text{ J} = \boxed{54\text{ J}}$$

Problem #2: Convert U_G to Heat and K

Blocks #1 and #2 are attached as shown, where block #1 is on a **rough, horizontal surface** with kinetic coefficient of friction μ_k. Block #2 will **fall a height *h*** in this problem, converting its gravitational potential energy into kinetic energy of the blocks and lost heat due to block #1 sliding on the surface with friction. Assume that the gravitational energy of block #2 is zero after it has fallen the distance *h*.

(a) Draw a **picture** of the problem and label the distance variables (*h* and Δs, where $\Delta s = h$).



(b) Using the Work-Energy Theorem #2, find an **algebraic expression** for the velocity *v* of the blocks after block #2 falls a distance *h* in terms of the other variables (m_1, m_2, h, μ_k). If the height ***h* is tripled**, how does the velocity *v* change?

$\Delta K + \Delta U + f\Delta s = W_{ext}$ where $W_{ext} = 0$ and $U_{1G} = 0$ (no height change)

$(K_{1f} - K_{1i}) + (K_{2f} - K_{2i}) + (U_{2Gf} - U_{2Gi}) + f\Delta s = 0$ where $f = \mu_k m_1 g$ and $\Delta s = h$

$\left(\tfrac{1}{2}m_1 v^2 - 0\right) + \left(\tfrac{1}{2}m_2 v^2 - 0\right) + \left(0 - m_2 gh\right) + \left(\mu_k m_1 g\right)h = 0$

$\tfrac{1}{2}(m_1 + m_2)v^2 = m_2 gh - \mu_k m_1 gh$

$\boxed{v = \sqrt{\dfrac{2gh(m_2 - \mu_k m_1)}{m_1 + m_2}}}$ If *h* is tripled, then *v* is increased by $\boxed{\sqrt{3}}$.

(c) Find the **numerical value** of the **velocity *v*** given the following: $m_1 = 2$ kg, $m_2 = 4$ kg, $h = 4$ m, $\mu_k = 0.5$. Use $g = 10$ m/s^2.

$$v = \sqrt{\frac{2gh(m_2 - \mu_k m_1)}{m_1 + m_2}} = \sqrt{\frac{2(10\text{ m/s}^2)(4\text{ m})(4\text{ kg} - 0.5(2\text{ kg}))}{2\text{ kg} + 4\text{ kg}}}$$

$\boxed{v = \sqrt{40} = 6.3\text{ m/s}}$

(d) Find the following **numerical energy values** using their definitions and values from part (c): 1) **initial potential** energy of block #2, 2) energy **dissipated** by block #1, and 3) total **final kinetic** energy of both blocks. Use $g = 10$ m/s^2 and round to the nearest Joule. Check that the initial potential energy of block #2 does in fact equal the dissipated energy plus the final total kinetic energy.

1) $U_{2Gi} = mgh = (4\text{ kg})(10\text{ m/s}^2)(4\text{ m}) = \boxed{160\text{ J}}$

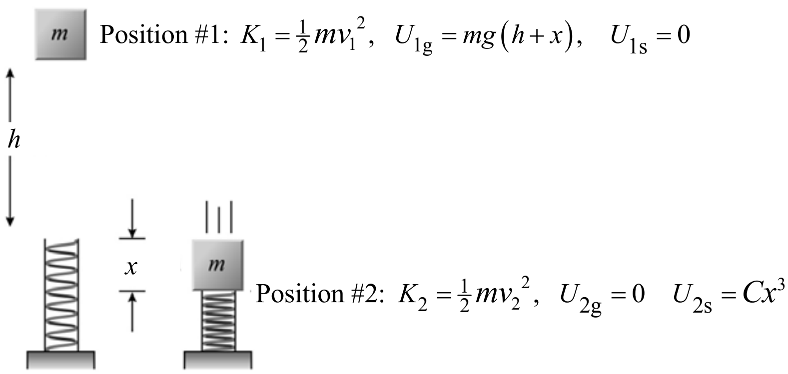
2) $f\Delta s = \mu_k mg\Delta s = 0.5(2\text{ kg})(10\text{ m/s}^2)(4\text{ m}) = \boxed{40\text{ J}}$

3) $K_{1f+2f} = \tfrac{1}{2}(m_1 + m_2)v^2 = \tfrac{1}{2}(2\text{ kg} + 4\text{ kg})\left(\sqrt{40}\text{ m/s}\right)^2 = \boxed{120\text{ J}}$

<u>Problem #3</u>: Convert U_G and K to U_S

A block of mass *m* is thrown vertically **UPWARDS** with an **initial vertical velocity** v_1. It is located at a **height *h*** (position #1) above the **<u>top</u>** of a "sponge cylinder". Assume that this cylinder is composed of a material such that when a mass pushes down on it, **potential energy** is stored and given by **$U = Cx^3$**, where C is a constant and *x* is the distance that the mass has compressed the cylinder. After being thrown, the block eventually comes into contact with the sponge cylinder and slows down, **compressing** the cylinder an **amount *x*** (position #2). <u>Assume</u> that the **gravitational** potential energy of the block is **zero** when it has compressed the spring by *x*.

(a) Draw a **picture** of the block and spring for the **initial** and **final** positions of the block and label the relevant **distances** (*h*, *x*).



(b) Using the **Work-Energy Theorem #2**, find an **algebraic expression** for the **velocity** v_2 of the block (position #2) after it compresses the cylinder by an amount *x*.
In this problem, the block's initial gravitational potential energy (position #1) and kinetic energy is converted into "sponge" potential energy and kinetic energy after compressing the spring (position #2). Note that the initial kinetic energy of the block only depends on the magnitude of its initial velocity (or speed), NOT its direction. So, the problem is identical whether the block is thrown upwards or downwards.

$$\boxed{\Delta K + \Delta U = 0} \quad \text{because } W_{\text{ext}} = 0 \text{ and } f = 0$$

$$(K_2 - K_1) + (U_{2g} - U_{1g})_{\text{gravity}} + (U_{2s} - U_{1s})_{\text{spring}} = 0$$

$$\left(\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2\right) + \left(0 - mg(h+x)\right) + \left(Cx^3 - 0\right) = 0$$

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}mv_1^2 + mg(h+x) - Cx^3$$

$$\boxed{v_2 = \sqrt{v_1^2 + 2g(h+x) - \frac{2Cx^3}{m}}}$$

(c) Find an **algebraic expression** for the **compression distance** *x* of the spring. To **simplify** solving for *x*, **ASSUME** that the height *h* is MUCH **greater** than x, i.e. $\boldsymbol{h + x \sim x}$.

$$(K_2 - K_1) + (U_{2g} - U_{1g})_{\text{gravity}} + (U_{2s} - U_{1s})_{\text{spring}} = 0$$

$$\left(\tfrac{1}{2}mv_2{}^2 - \tfrac{1}{2}mv_1{}^2\right) + (0 - mgh) + \left(Cx^3 - 0\right) = 0$$

$$x^3 = \frac{m}{2C}\left(v_1{}^2 - v_2{}^2\right) + \frac{mgh}{C}$$

$$\boxed{x = \left[\frac{m}{2C}\left(v_1{}^2 - v_2{}^2\right) + \frac{mgh}{C}\right]^{\frac{1}{3}}}$$

(d) Find the **numerical value** of the compression **distance** ***x*** using part (c) and the following values: $m = 2$ kg, $h = 5$ m, $v = 3$ m/s, $C = 5000$ N/m^2. Use $g = 10$ m/s^2. Check the assumption that *x* is much smaller than *h*.

$$x = \left[\frac{m}{2C}\left(v_1{}^2 - v_2{}^2\right) + \frac{mgh}{C}\right]^{\frac{1}{3}}$$

$$x = \left[\frac{2\text{ kg}}{2\left(5000\text{ N/m}^2\right)}\left((3\text{ m/s})^2 - 0\right) + \frac{(2\text{ kg})\left(10\text{ m/s}^2\right)(5\text{ m})}{\left(5000\text{ N/m}^2\right)}\right]^{\frac{1}{3}}$$

$\boxed{x = 0.28\text{ m}}$ = 5.6% of height *h*

(e) Find the following **numerical energy values**: 1) **initial** energy of the block (gravitational potential and kinetic), and 2) **final** spring potential energy of the block. Assume $m = 2$ kg, $h = 5$ m, $v = 3$ m/s, and $C = 5000$ N/m^2.Use $g = 10$ m/s^2 and round to the nearest Joule. Compare the initial and final energies to check if they are within 1%.

1) $U_{\text{Gi}} = mgh = (2\text{ kg})\left(10\text{ m/s}^2\right)(5\text{ m}) = 100\text{ J}$

$K_{\text{i}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2\text{ kg})(3\text{ m/s})^2 = 9\text{ J}$

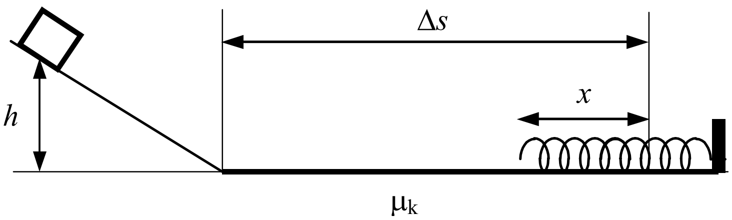
$\boxed{E_{\text{Tot i}}} = U_{\text{Gi}} + K_{\text{i}} = 100\text{ J} + 9\text{ J} = \boxed{109\text{ J}}$

2) $\boxed{U_{\text{Sf}}} = Cx^3 = (5000\text{ N/m})(0.28\text{ m})^3 = \boxed{110\text{ J}}$ Initial and final energies are within 1%.

<u>Problem #4</u>: Convert U_G to Heat and U_S

A block starts from rest and slides down a **frictionless ramp** of height h. The block then slides a total distance Δs on a **rough horizontal surface** with kinetic coefficient of friction μ_k. At the **<u>end</u>** of its travel, the block **compresses a horizontal spring** with force constant k by a distance x. (Note: The distance x is included in the total distance Δs on the rough surface.)

(a) Draw a **picture** of the problem and label the distance variables (h, Δs, x).



(b) Using the Work-Energy Theorem #2, find an **algebraic expression** for the spring compression **distance *x*** in terms of the other variables (m, h, μ_k, Δs, k).

$$\Delta K + \Delta U + f\Delta s = W_{\text{ext}} \quad \text{where } W_{\text{ext}} = 0$$

$$(K_f - K_i) + (U_{\text{Gf}} - U_{\text{Gi}}) + (U_{\text{Sf}} - U_{\text{Si}}) + f\Delta s = 0 \quad \text{where } f = \mu_k mg$$

$$(0-0) + (0 - mgh) + \left(\tfrac{1}{2}kx^2 - 0\right) + (\mu_k mg)\Delta s = 0$$

$$\tfrac{1}{2}kx^2 = mgh - \mu_k mg\Delta s$$

$$\boxed{x = \sqrt{\frac{2mg}{k}(h - \mu_k \Delta s)}}$$

(c) Find the **numerical value** of the compression **distance *x*** given the following: m = 4 kg, h = 4 m, μ_k = 0.5, Δs = 2 m, and k = 1000 N/m. Use g = 10 m/s^2.

$$x = \sqrt{\frac{2mg}{k}(h - \mu_k \Delta s)} = \sqrt{\frac{2(4\text{ kg})(10\text{ m/s}^2)(4\text{ m} - 0.5(2\text{ m}))}{(1000\text{ N/m})}}$$

$$\boxed{x = 0.49\text{ m}}$$

(d) Find the following **numerical energy values** using their definitions and values from part (c): 1) **initial** gravitational potential energy, 2) energy **dissipated** by heat, and 3) **final** spring potential energy. Use g = 10 m/s^2 and round to the nearest Joule. Check that the initial gravitational potential energy equals the dissipated energy plus the final spring potential energy.

1) $U_{\text{Gi}} = mgh = (4\text{ kg})(10\text{ m/s}^2)(4\text{ m}) = \boxed{160\text{ J}}$

2) $f\Delta s = \mu_k mg\Delta s = 0.5(4\text{ kg})(10\text{ m/s}^2)(2\text{ m}) = \boxed{40\text{ J}}$

3) $U_{\text{Sf}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(1000\text{ N/m})(0.49\text{ m})^2 = \boxed{120\text{ J}}$