

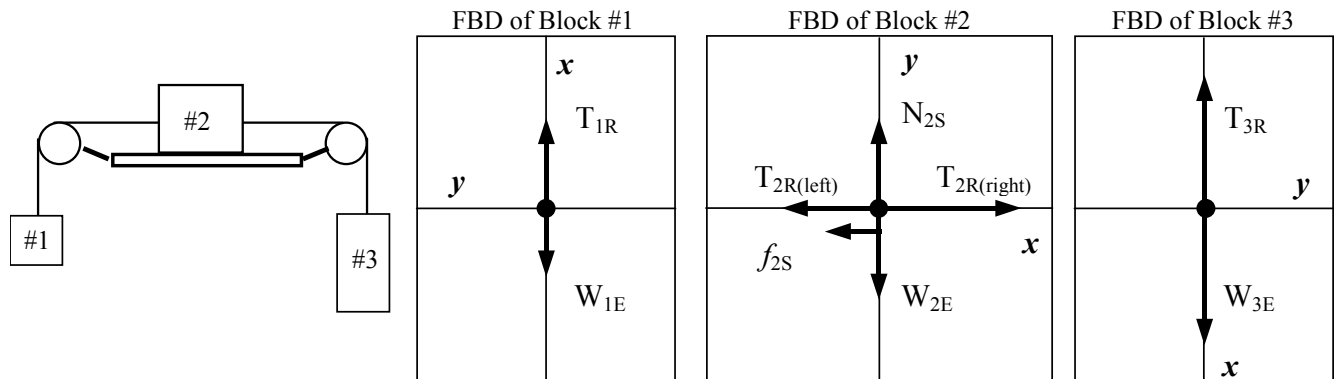
HOMEWORK Solutions #4: Forces II (Phys 207, Fall 2005)

QUIZ #4 on HW#4 THIS Thursday (9/29/05) at BEGINNING of class

Problem #1: Moving Horizontal and Vertical Blocks with Friction

Three blocks (masses m_1, m_2, m_3) are connected by two massless strings that slide over frictionless pegs as shown below. There is a coefficient of kinetic friction μ_k between block #2 and the horizontal surface. Assume that Block #3 is heavier than Block #1 and the blocks accelerate such that Block #1 rises and Block #3 falls.

- (a) Draw the **free body diagrams** for blocks #1, #2, and #3 in the indicated boxes. Use **SUBSCRIPT notation** to label ALL vectors with appropriate labels as done in class, e.g. $T_{2R(left)}$, $T_{2R(right)}$, f_{2S} , etc. Mark on the axes drawn in the boxes where the **+x and +y axes** are located.



- (b) Write down $\sum F = ma$ for blocks #1, #2, and #3 using the **notation in the FBD's**.

$$\sum F_{1x} = T_{1R} - W_{1E} = m_1 a \quad (\text{Eqn. \#1})$$

$$\sum F_{2x} = T_{2R(right)} - T_{2R(left)} - f_{2S} = m_2 a \quad (\text{Eqn. \#2})$$

$$\sum F_{2y} = N_{2S} - W_{2E} = 0$$

$$\sum F_{3x} = W_{3E} - T_{3R} = m_3 a \quad (\text{Eqn. \#3})$$

- (c) Use the force equations in part (b) to find the **acceleration a** of the blocks in terms of the **notation in the FBD's**.

>> Add Eqns. #1, #2, and #3 from part (b) and solve for a .

$$(T_{1R} - W_{1E}) + (T_{2R(right)} - T_{2R(left)} - f_{2S}) + (W_{3E} - T_{3R}) = m_1 a + m_2 a + m_3 a$$

where $T_{1R} = T_{2R(left)}$, $T_{3R} = T_{2R(right)}$

$$a = \frac{W_{3E} - W_{1E} - f_{2S}}{m_1 + m_2 + m_3}$$

- (d) Express the **friction f_{2S}** using the **given variables** m_1, m_2, μ_k , and g . and then rewrite the algebraic expression for the **acceleration a** in terms of the given variables.

$$f_{2S} = \mu_k N_{2S} = \mu_k W_{2E} \quad \text{using } \sum F_{2y}$$

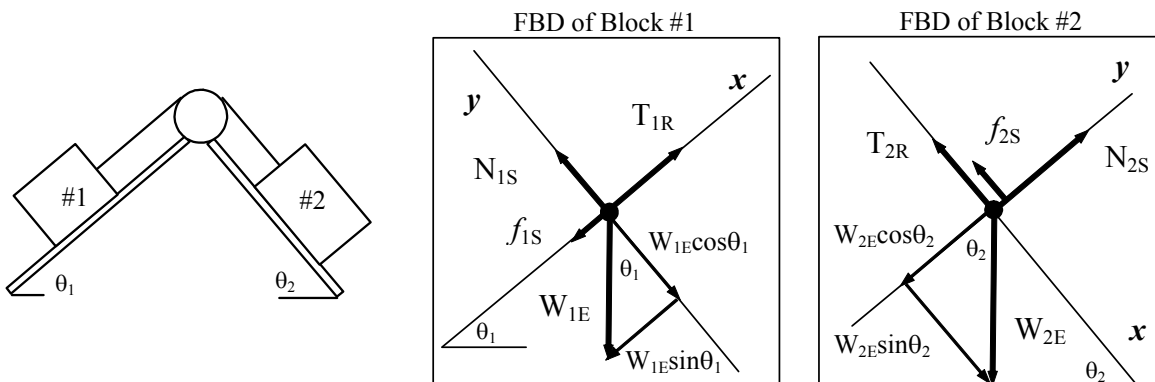
$$f_{2S} = \mu_k m_2 g$$

$$a = \frac{m_3 g - m_1 g - \mu_k m_2 g}{m_1 + m_2 + m_3}$$

Problem #2: Moving Blocks on Inclined Planes with Friction

Two blocks (masses m_1, m_2) are connected by a massless string that slides over a frictionless peg as shown below. Both blocks are on inclines (angles θ_1, θ_2) with coefficient of kinetic friction μ_k . Assume that Block #2 is heavier than Block #1 and the blocks accelerate such that Block #1 rises and Block #2 falls.

- (a) Draw the **free body diagrams** for blocks #1 and #2 in the indicated boxes. Use **SUBSCRIPT notation** to label ALL vectors with appropriate labels as done in class, e.g. T_{1R}, f_{1S} , etc. Mark on the axes drawn in the boxes where the **+x and +y axes** are located. Also, draw and label the x- and y-component vectors for any forces not aligned along the indicated axes, e.g., W_{1E} and W_{2E} .



- (b) Write down $\sum F = ma$ for blocks #1 and #2 using the **notation in the FBD's**.

$$\sum F_{1x} = T_{1R} - W_{1E} \sin \theta_1 - f_{1S} = m_1 a \quad (\text{Eqn. \#1})$$

$$\sum F_{1y} = N_{1S} - W_{1E} \cos \theta_1 = 0$$

$$\sum F_{2x} = W_{2E} \sin \theta_2 - T_{2R} - f_{2S} = m_2 a \quad (\text{Eqn. \#2})$$

$$\sum F_{2y} = N_{2S} - W_{2E} \cos \theta_2 = 0$$

- (c) Use the force equations in part (b) to find the **acceleration a** of the blocks in terms of the **notation in the FBD's**.

>> Add Eqns. #1 and #2 from part (b) and solve for a .

$$(T_{1R} - W_{1E} \sin \theta_1 - f_{1S}) + (W_{2E} \sin \theta_2 - T_{2R} - f_{2S}) = m_1 a + m_2 a \quad \text{where } T_{1R} = T_{2R}$$

$$-W_{1E} \sin \theta_1 - f_{1S} + W_{2E} \sin \theta_2 - f_{2S} = (m_1 + m_2) a$$

$$a = \frac{-W_{1E} \sin \theta_1 - f_{1S} + W_{2E} \sin \theta_2 - f_{2S}}{m_1 + m_2}$$

- (d) Express the **frictions f_{1S} and f_{2S}** using the given variables $m_1, m_2, \mu_k, \theta_1, \theta_2$, and g .

$$f_{1S} = \mu_k N_{1S} = \mu_k W_{1E} \cos \theta_1 \quad \text{using } \sum F_{1y}$$

$$f_{1S} = \mu_k m_1 g \cos \theta_1$$

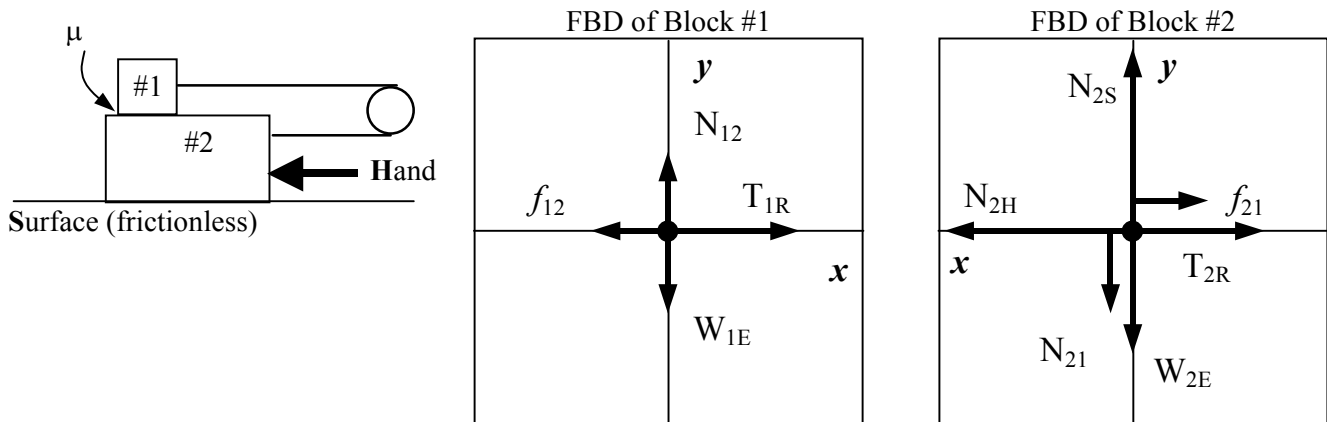
$$f_{2S} = \mu_k N_{2S} = \mu_k W_{2E} \cos \theta_2 \quad \text{using } \sum F_{2y}$$

$$f_{2S} = \mu_k m_2 g \cos \theta_2$$

Problem #3: Moving Horizontal Blocks with Friction between them

Two blocks (masses m_1, m_2) are connected by a massless string that slides over a frictionless peg as shown below. There is no friction between block #2 and the bottom horizontal surface, but there is a coefficient of kinetic friction μ_k between the two blocks. A hand pushes block #2 to the left with a normal force.

- (a) Draw the **free body diagrams** for blocks #1 and #2 in the indicated boxes. Use **SUBSCRIPT notation** to label ALL vectors with appropriate labels as done in class, e.g. W_{2E}, f_{21} , etc. Mark on the axes drawn in the boxes where the **+x and +y axes** are located.



- (b) Write down $\sum F = ma$ for blocks #1 and #2 using the **notation in the FBD's**.

$$\sum F_{1x} = T_{1R} - f_{12} = m_1 a \quad (\text{Eqn. \#1})$$

$$\sum F_{1y} = N_{12} - W_{1E} = 0$$

$$\sum F_{2x} = N_{2H} - T_{2R} - f_{21} = m_2 a \quad (\text{Eqn. \#2})$$

$$\sum F_{2y} = N_{2S} - N_{21} - W_{2E} = 0$$

- (c) Use the force equations in part (b) to find the **acceleration a** of the blocks in terms of the **notation in the FBD's**.

>> Add Eqns. #1 and #2 from part (b) and solve for a .

$$(T_{1R} - f_{12}) + (N_{2H} - T_{2R} - f_{21}) = m_1 a + m_2 a$$

where $T_{1R} = T_{2R}$ and $f_{12} = f_{21}$

$$N_{2H} - 2f_{12} = m_1 a + m_2 a$$

$$a = \frac{N_{2H} - 2f_{12}}{m_1 + m_2}$$

- (d) Express the **friction f_{12}** using the given variables m_1, m_2, μ_k , and/or g .

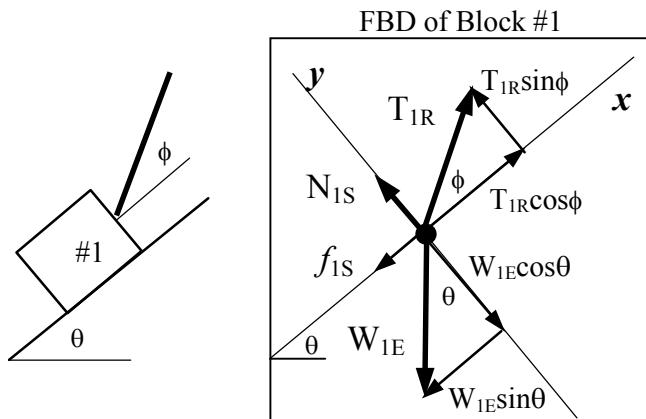
$$f_{12} = \mu_k N_{12} = \mu_k W_{1E} \quad \text{using } \sum F_{1y}$$

$$f_{12} = \mu_k m_1 g$$

Problem #4: Pulled Block on Incline with Friction

Block #1 (mass m_1) is pulled by a rope up an incline (angle θ) with a coefficient of kinetic friction μ_k . The rope is pulled with a tension T_{1R} at an angle ϕ with respect to the incline.

- (a) Draw the **free body diagram** for block #1 in the indicated box. Use **SUBSCRIPT notation** to label ALL vectors with appropriate labels as done in class, e.g. W_{1E} , T_{1R} , etc. Mark on the axes drawn in the boxes where the **+x and +y axes** are located. Also, draw and label the x - and y -component vectors for any forces not aligned along the indicated axes, e.g. W_{1E} and T_{1R} .



- (b) Write down $\sum F = ma$ for block #1 using the **notation in the FBD**.

$$\sum F_{1x} = T_{1R} \cos \phi - W_{1E} \sin \theta - f_{1S} = m_1 a \quad (\text{Eqn. \#1})$$

$$\sum F_{1y} = N_{1S} + T_{1R} \sin \phi - W_{1E} \cos \theta = 0$$

- (c) Express the **friction** f_{1S} using the **given variables** T_{1R} , m_1 , μ_k , θ , ϕ , and g .

$$f_{1S} = \mu_k N_{1S} = \mu_k (W_{1E} \cos \theta - T_{1R} \sin \phi) \quad \text{using } \sum F_{1y}$$

$$f_{1S} = \mu_k (m_1 g \cos \theta - T_{1R} \sin \phi)$$

- (d) Use the force equations in part (b) to find the **acceleration** a of the block in terms of the **given variables** T_{1R} , m_1 , μ_k , θ , ϕ , and g .

$$a = \frac{T_{1R} \cos \phi - W_{1E} \sin \theta - f_{1S}}{m_1}$$

$$a = \frac{T_{1R} \cos \phi - m_1 g \sin \theta - \mu_k (m_1 g \cos \theta - T_{1R} \sin \phi)}{m_1}$$