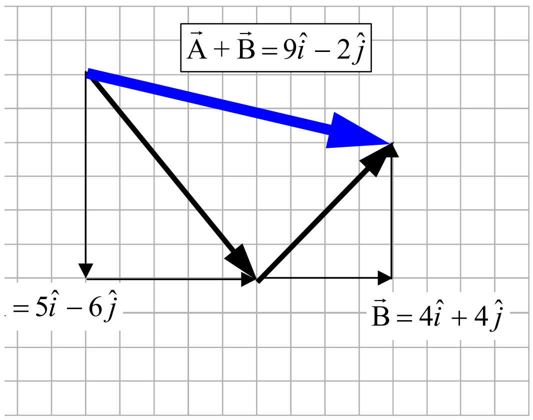
HW#2 Solutions: Motion in 2-D (Phys 207, Fall 2005)

QUIZ #2 on HW#2 THIS Thursday (9/15) at BEGINNING of class

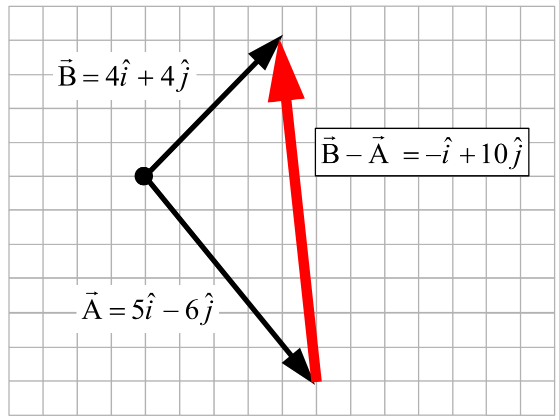
Problem #1: Vector Addition and Subtraction

(a) Show on the grids below the graphical representation of $\vec{A}+\vec{B}$ (use head to tail) and $\vec{B}-\vec{A}$ (tail to tail) for: $\vec{A}=5\hat{\imath}-6\hat{\jmath}$ and $\vec{B}=4\hat{\imath}+4\hat{\jmath}$.

$\vec{A}+\vec{B}$



$\vec{B}-\vec{A}$



(b) Find the magnitude (length) of $\vec{A}$, $\vec{B}$, $\vec{A}+\vec{B}$, and $\vec{B}-\vec{A}$.

$\vec{A}=5\hat{\imath}-6\hat{\jmath}$ $\vec{B}=4\hat{\imath}+4\hat{\jmath}$

$$\boxed{\left|\vec{A}\right|}=\sqrt{A_x^2+A_y^2}=\sqrt{(5)^2+(-6)^2}=\boxed{\sqrt{61}\text{ or }7.8}$$

$$\boxed{\left|\vec{B}\right|}=\sqrt{B_x^2+B_y^2}=\sqrt{(4)^2+(4)^2}=\boxed{\sqrt{32}\text{ or }5.7}$$

$$\boxed{\vec{A}+\vec{B}}=(5+4)\hat{\imath}+(-6+4)\hat{\jmath}=9\hat{\imath}-2\hat{\jmath}$$

$$\left|\vec{A}+\vec{B}\right|=\sqrt{(9)^2+(-2)^2}=\boxed{\sqrt{85}\text{ or }9.2}$$

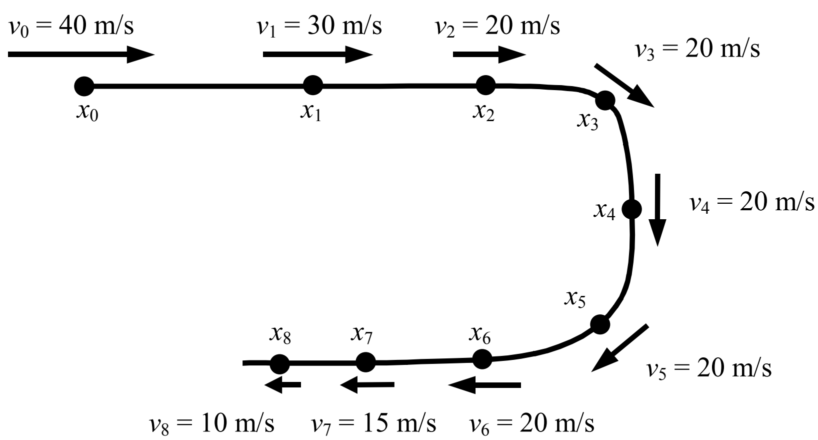
$$\boxed{\vec{B}-\vec{A}}=(4-5)\hat{\imath}+(4+6)\hat{\jmath}=-\hat{\imath}+10\hat{\jmath}$$

$$\left|\vec{B}-\vec{A}\right|=\sqrt{(-1)^2+(10)^2}=\boxed{\sqrt{101}\text{ or }10}$$

Problem #2: Velocity and Acceleration Vectors

In the path shown below, the locations of a car driving around a track are shown at EQUAL time intervals where $\mathbf{\Delta t = 5}$ **s**. The car starts with speed **40 m/s at x_0** and uniformly **slows down** to 20 m/s from x_0 to x_2. The car then travels at a **constant** speed from x_2 to x_6. The first corner at x_3 has a smaller radius curve (r_1 = 50 m) and the second corner at x_5 has a larger radius curve (r_2 = 100 m). At **point** x_6, the car starts to **slow down** and is traveling at 10 m/s at point x_8. Assume that the car continues to slow down after point x_8.

(a) On the first drawing, *carefully* draw the **INSTANTANEOUS velocity vectors** v_0 to v_8 of the car at points x_0 to x_8 around the path. Show both an appropriate length and direction for each vector**,** and **label** the **magnitude** of the vectors v_0 to v_8.



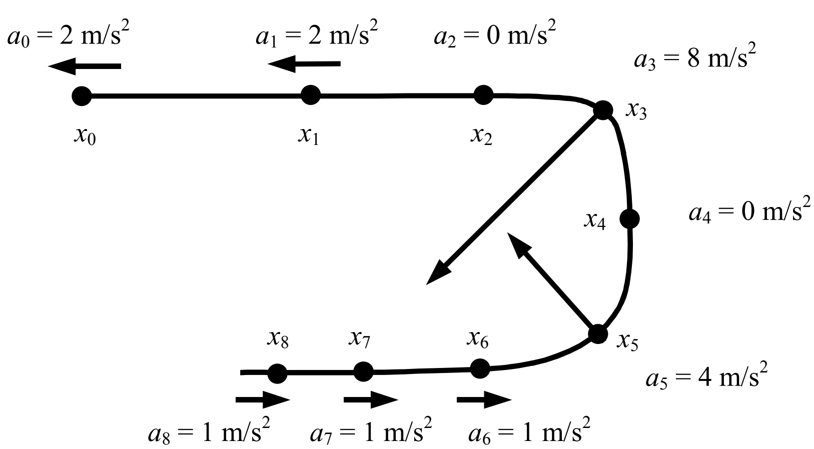
(b) Calculate the **magnitude** (no plus or minus signs) of the **INSTANTANEOUS accelerations** a_0 to a_8 at the points x_0 to x_8 around the path.

$$\boxed{\left|a_{0,1}\right|} = \left|\frac{\Delta v}{\Delta t}\right| = \left|\frac{20-40\text{ m/s}}{10\text{ s}}\right| = \boxed{2\text{ m/s}^2} \quad \text{(car speeding up);} \qquad \boxed{a_{2,4} = 0\text{ m/s}^2} \quad \text{(car "coasting")}$$

$$\text{Centripetal Acceleration: } \boxed{a_3} = \frac{v^2}{r} = \frac{(20\text{ m/s})^2}{50\text{ m}} = \boxed{8\text{ m/s}^2}; \quad \boxed{a_5} = \frac{(20\text{ m/s})^2}{100\text{ m}} = \boxed{4\text{ m/s}^2}$$

$$\boxed{\left|a_{6,7,8}\right|} = \left|\frac{\Delta v}{\Delta t}\right| = \left|\frac{10-20\text{ m/s}}{10\text{ s}}\right| = \boxed{1\text{ m/s}^2} \quad \text{(car slowing down)}$$

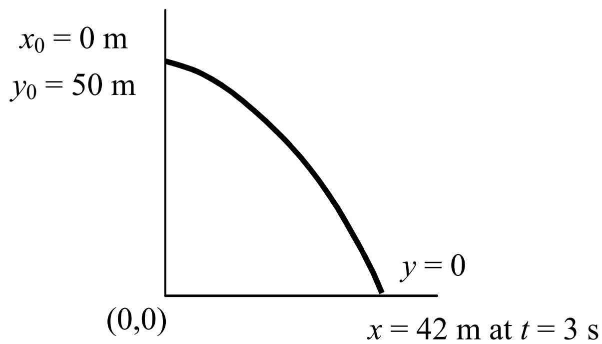
(c) On the second drawing, draw the **INSTANTANEOUS acceleration vectors** of the car at points x_0 to x_8**.** Show both an appropriate length and direction for each vector**,** and **label** the **magnitude** of the vectors a_0 to a_8.



Problem #3: Initial Velocity for Projectile Motion

Using a super slingshot, a troublemaker shoots a rock downward from a window located **50 m** above the ground. After flying through the air for **3 s**, the rock barely misses a person on the ground and lands **42 m** away from the bottom of the building.

(a) Draw a **picture** of the problem with the origin located on the ground below where the rock is shot. Draw the **x-y axes** with the **origin** labeled (0,0), sketch the **path** of the rock, and indicate ALL known **values**, e.g. x_0, y_0, x & y at $t = 3$ s.



(b) Find an algebraic expression for the **initial horizontal velocity** $\boldsymbol{v_{x0}}$ of the rock and then find its numerical value.

$$\boxed{v_{0x} = v_x = \frac{\Delta x}{\Delta t}} = \frac{42 \text{ m}}{3 \text{ s}} = \boxed{14 \text{ m/s}}$$

(c) Find an algebraic expression for the **initial vertical velocity** $\boldsymbol{v_{y0}}$ of the rock and then find its numerical value. Note: Start from the y-equation for projectile motion.

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$\boxed{v_{0y} = \frac{(y - y_0) + \frac{1}{2}gt^2}{t}}$$

$$v_{0y} = \frac{(0 \text{ m} - 50 \text{ m}) + \frac{1}{2}(9.8 \text{ m/s}^2)(3 \text{ s})^2}{(3 \text{ s})} = \boxed{-2.0 \text{ m/s}}$$

(d) Find the **magnitude and angle of the total velocity** $\boldsymbol{v_0}$ using t he Pythagorean theorem and tangent relationship.

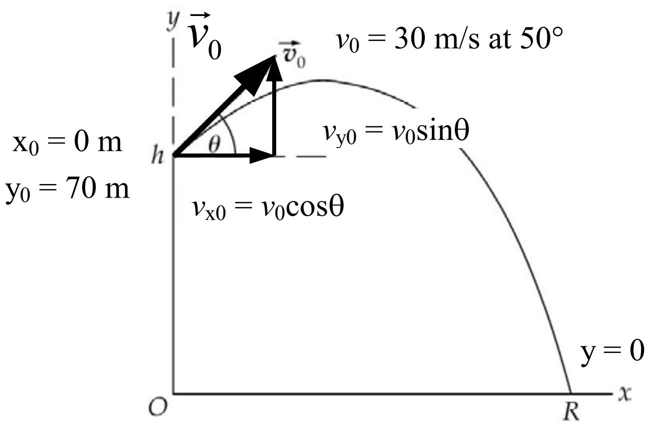
$$\boxed{v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}} = \sqrt{(14 \text{ m/s})^2 + (2 \text{ m/s})^2} = \boxed{14 \text{ m/s}}$$

$$\boxed{\theta = \tan^{-1}\frac{v_{0y}}{v_{0x}}} = \tan^{-1}\left(\frac{-2 \text{ m/s}}{14 \text{ m/s}}\right) = \boxed{-8.1^\circ} \text{ (below horizontal)}$$

Problem #4: Flight Time and Range for Projectile Motion

A mini-rocket is shot from the roof of a building that is **70 m tall**. It is launched at an **angle of 50°** above the horizontal with an initial speed of **30 m/s**.

(a) Draw a **picture** of the problem with the origin located on the ground below where the rocket is shot. Draw the **x-y axes** with the **origin** labeled (0,0), sketch the **path** of the rocket, and indicate ALL known **values**, e.g. x_0, y_0, and v_0.



(b) Write the ***x*- and y-equations** of the rocket in algebraic form AND then rewrite them with any known values substituted.

$x = v_{xo}t$ where $v_{xo} = (30\text{ m/s})\cos 50° = 19\text{ m/s}$

$\boxed{x = (19\text{ m/s})t}$

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = (30\text{ m/s})\sin 50° = 23\text{ m/s}$

$\boxed{y = (70\text{ m}) + (23\text{ m/s})t - (4.9\text{ m/s}^2)t^2}$

(c) Find the **flight time *t*** of the rocket. Remember that this is the time solution for the *y*-equation when the projectile returns to the ground, i.e. when $y = 0$.

$$-(4.9\text{ m/s}^2)t^2 + (23\text{ m/s})t + (70\text{ m}) = 0$$

$$t = \frac{-(23\text{ m/s}) \pm \sqrt{(23\text{ m/s})^2 + 4(4.9\text{ m/s}^2)(70\text{ m})}}{-2(4.9\text{ m/s}^2)}$$

$\boxed{t} = 2.35 \pm 4.45 = -2.1\text{ s}$ or $\boxed{6.8\text{ s}}$

(d) Find the **range *x*** of the rocket using the *x*-equation and flight time.

$\boxed{x = v_{0x}t} = (19\text{ m/s})(6.8\text{ s}) = \boxed{129\text{ m}}$