<u>HW#1 Solutions</u>: Motion in 1-D (Phys 207, Fall 2005)

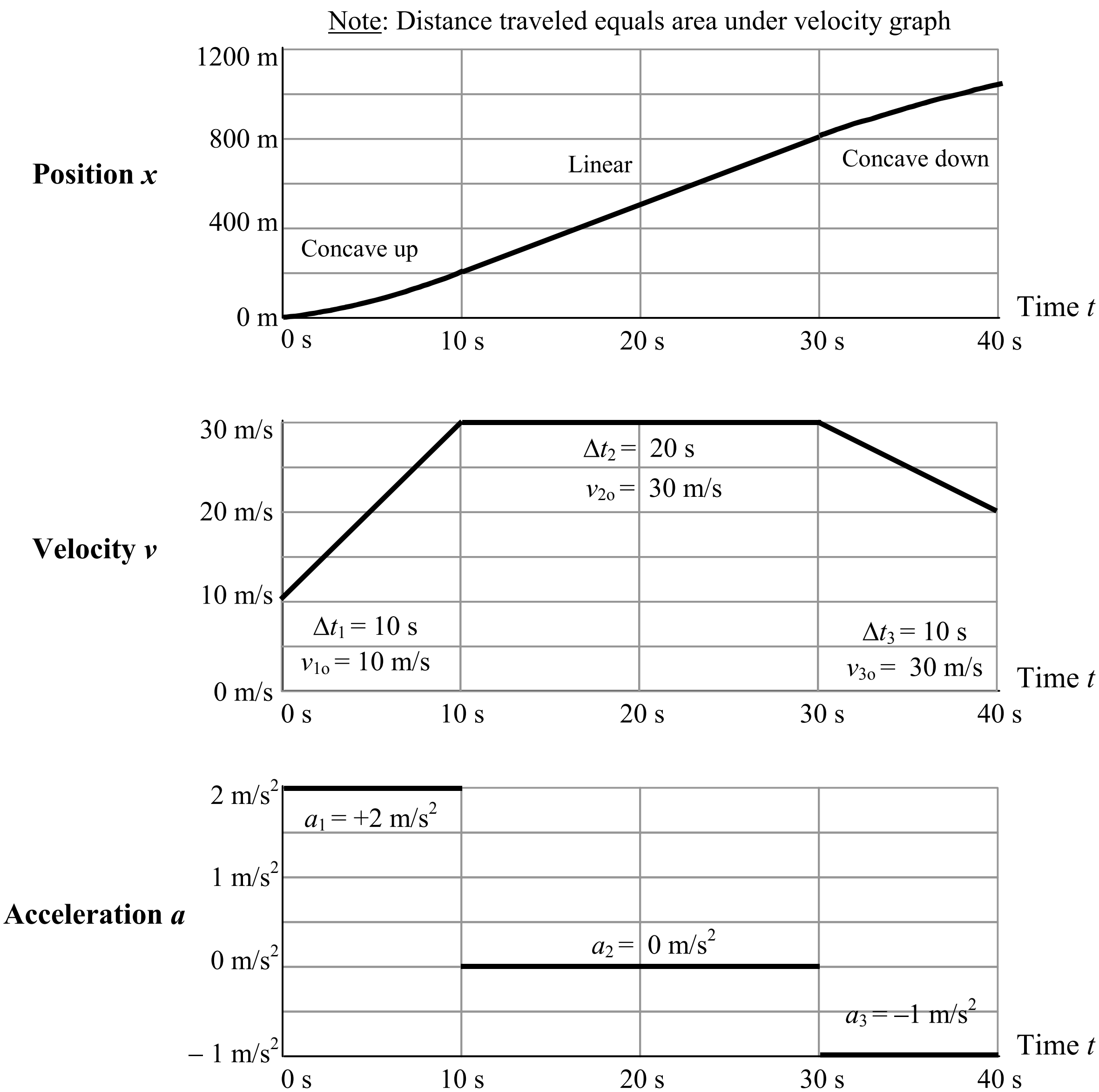
QUIZ #1 on HW#1 THIS Thursday (Sept. 8) at BEGINNING of class

<u>Problem #1</u>: Motion of Car with Different Constant Accelerations

While driving your car, you are merging onto a highway and at time $t = 0$ **speed up** from 10 m/s to 30 m/s over 10 s. You then **drive** for 20 s before encountering a construction zone. You then **slow down** at 1 m/s^2 until reaching the "safe speed" of 20 m/s.

(a) **<u>Accurately</u>** draw the **position** (x), **velocity** (v), and **acceleration** (a) vs. time **graphs** for your car's motion from the time you start speeding up until the time you reach the safe speed. Arrange the graphs vertically on the page with the position graph located at the top and the acceleration graph located at the bottom (see recitation sheet on 8/28 for an example). Graph paper is recommended.

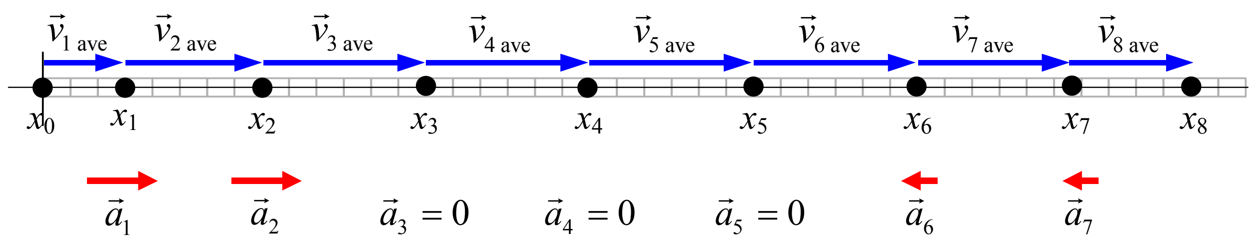
<u>**Note**</u>: It is usually easiest to begin with the velocity vs. time graph and then take its derivative for the acceleration vs. time graph, and its integral for the position vs. time graph.



(b) Draw the corresponding **motion diagram** for your car's motion with *x*-positions marked and labeled every 5 s. Label all average *v* vectors between the *x*-positions and the corresponding *a* vectors (see recitation sheet on 8/28 for an example).

Motion Diagram

x positions are separated by 5 sec intervals.



(c) Calculate the **total distance *x*** traveled from $t = 0$ when you started speeding up to when you reached the "safe speed." In your work, clearly show the distances (Δx_1, Δx_2, Δx_3) for each of the time periods (#1 = speed up, #2 = constant speed, #3 = slow down).

Note: In this problem, the distance traveled in **each time interval** corresponds to the area under the graph for that interval. The solution below shows the algebraic expressions for each interval, where the initial x_o values are zero and the distances are calculated from the velocity and acceleration terms. These answers are consistent with the geometric calculations for the areas under the graph in each interval.

$$\boxed{\Delta x_1} = v_{1o}\Delta t_1 + \tfrac{1}{2}a_1(\Delta t_1)^2 \quad \text{where } \Delta t_1 = 10\text{ s},\ a_1 = 2\text{ m/s}^2,\ v_{1o} = 10\text{ m/s}$$

$$\Delta x_1 = (10\text{ m/s})(10\text{ s}) + \tfrac{1}{2}(2\text{ m/s}^2)(10\text{ s})^2 = \boxed{200\text{ m}}$$

$$\boxed{\Delta x_2} = v_{2o}\Delta t_2 + \tfrac{1}{2}a_2(\Delta t_2)^2 \quad \text{where } \Delta t_2 = 20\text{ s},\ a_2 = 0,\ v_{2o} = v_{1o} + a_1\Delta t_1 = 10\text{ m/s} + (2\text{ m/s}^2)(10\text{ s}) = 30\text{ m/s}$$

$$\Delta x_2 = (30\text{ m/s})(20\text{ s}) = \boxed{600\text{ m}}$$

$$\boxed{\Delta x_3} = v_{3o}\Delta t_3 + \tfrac{1}{2}a_3(\Delta t_3)^2 \quad \text{where } \Delta t_3 = 10\text{ s},\ v_{3o} = v_{2o},\ a_3 = \frac{\Delta v_{3o}}{\Delta t_3} = \frac{-10\text{ m/s}}{10\text{ s}} = -1\text{ m/s}^2$$

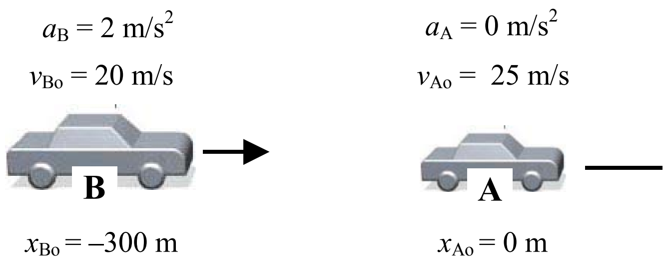
$$\Delta x_3 = (30\text{ m/s})(10\text{ s}) + \tfrac{1}{2}(-1\text{ m/s}^2)(10\text{ s})^2 = \boxed{250\text{ m}}$$

$$\boxed{\Delta x} = \Delta x_1 + \Delta x_2 + \Delta x_3 = 200\text{ m} + 600\text{ m} + 250\text{ m} = \boxed{1050\text{ m}}$$

Problem #2: Motion of Two Cars

You and a friend are driving your cars along a straight road. You (car A) are traveling at a constant speed of 25 m/s. Your friend (car B) is 300 m behind you and starts to accelerate at 2 m/s^2 from a speed of 20 m/s.

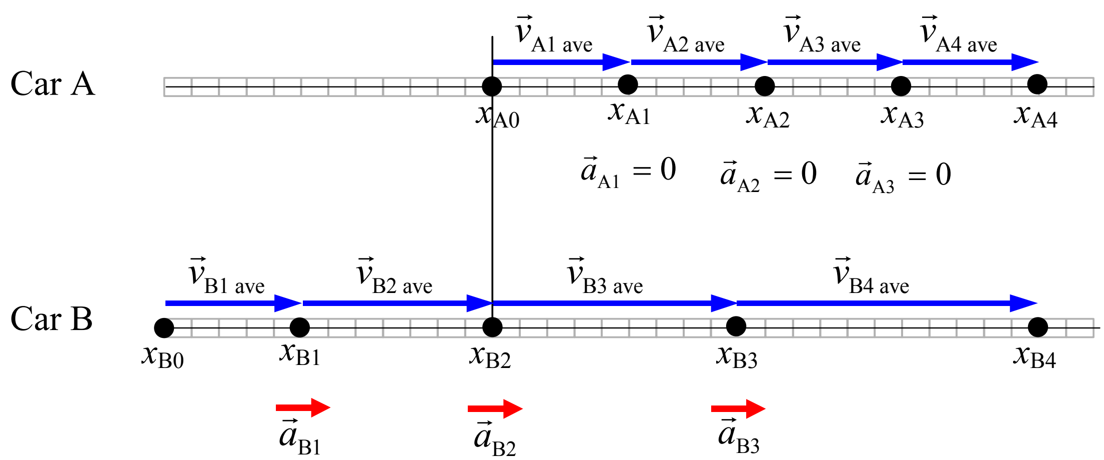
(a) Draw an initial **"picture"** of the problem with all the relevant variables and their numerical values indicated next to each car, e.g. accelerations a_A and a_B, initial velocities v_{A0} and v_{B0}, and initial locations x_{A0} and x_{B0}. Assume that you are located at the origin and your friend is at x_{B0} = –300 m.



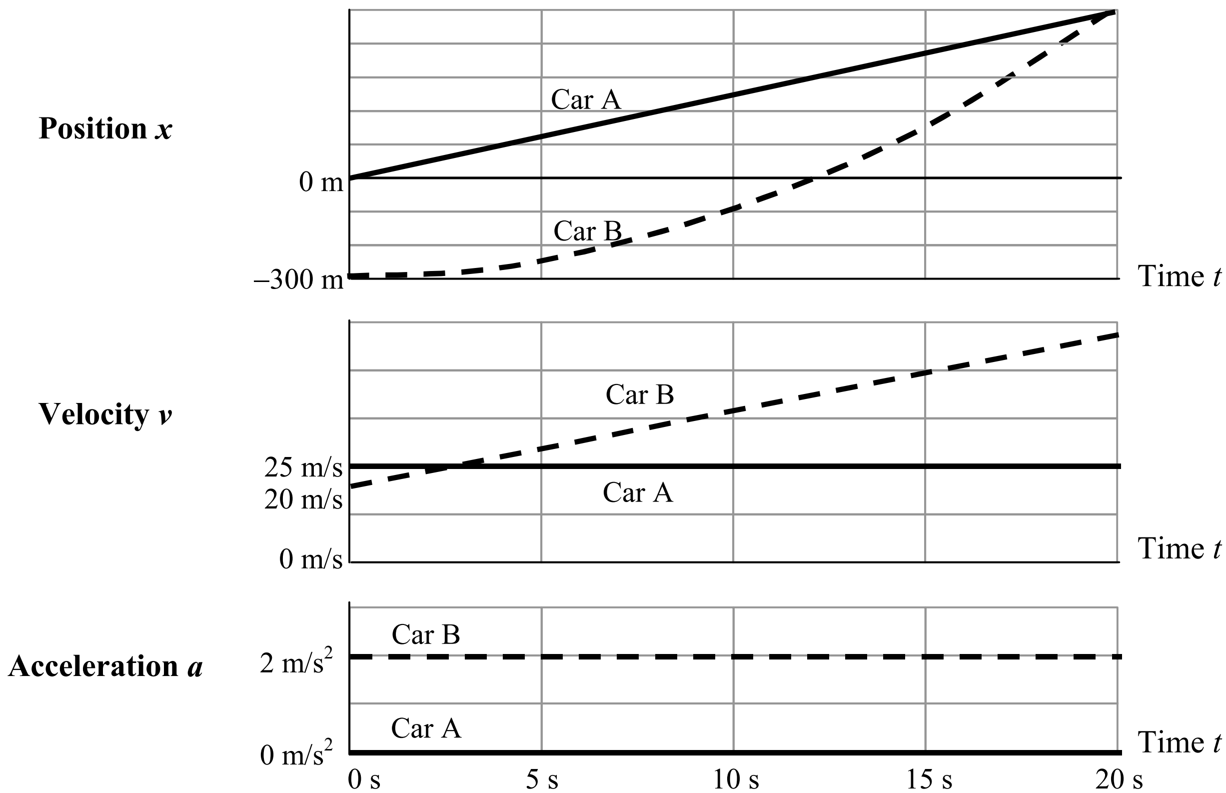
(b) Draw a **motion diagram** of the problem from when your friend starts accelerating to when he catches up with you. Draw and label at least (5) x-position "dots" with TWO DIFFERENT colors for Car A and Car B. Draw and label the average velocity vectors between the x-positions and indicate the uniform acceleration for each car.

Motion Diagram

x positions separated by 5 sec intervals.



(c) Sketch the **position** (x), **velocity** (v), and **acceleration** (a) vs. time **graphs** for both cars for the motion indicated in part (b). Arrange the three graphs vertically on the page with the position graph located at the top and the acceleration graph located at the bottom (see recitation sheet on 8/28 for an example).



(d) Write the x-equations for you (car A) and your friend (car B) with the initial position and velocity values substituted.

$$x_A = x_{Ao} + v_{Ao}t + \tfrac{1}{2}a_A t^2 \qquad x_B = x_{Bo} + v_{Bo}t + \tfrac{1}{2}a_B t^2$$

$$x_A = 0\text{ m} + (25\text{ m/s})t\ + 0\text{ m/s}^2 \qquad x_B = -300\text{ m} + (20\text{ m/s})t + \tfrac{1}{2}(2\text{ m/s}^2)t^2$$

$$\boxed{x_A = (25\text{ m/s})t} \qquad \boxed{x_B = -300\text{ m} + (20\text{ m/s})t + (1\text{ m/s}^2)t^2}$$

(e) Using the equations from part (d), find the time at which your friend "catches up" to you.

To find when the cars are at the same location, set x_A equal to x_B and solve for the time t.

$$x_A = x_B$$

$$(25\text{ m/s})t = -300\text{ m} + (20\text{ m/s})t + (1\text{ m/s}^2)t^2$$

$$t^2 - 5t - 300 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{5^2 - 4(1)(-300)}}{2(1)} = \frac{5 \pm 35}{2}$$

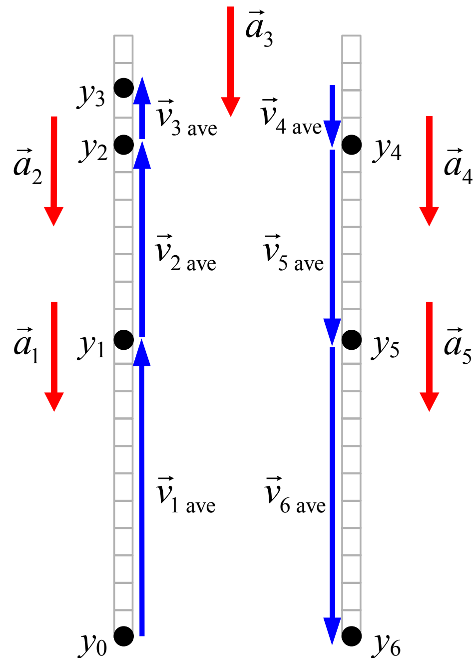
$\boxed{t = 20\text{ s}}$ (Note: Choose the positive root for the time solution.)

Problem #3: Vertical Motion of a Thrown Object

At time $t = 0$, a ball is thrown upward from the ground with an initial velocity of 20 m/s.

Motion Diagram

y positions separated by 2/3 sec intervals



(a) Draw the **motion diagram** for the ball's motion with at least (7) y-positions marked and labeled. Label all average v vectors between the y-positions and the corresponding a vectors (see p. 20 in book for an example)

(b) Find the time t at which the ball reaches its apex (i.e., highest point). Show all work algebraically and then substitute numerical values at the end.

Note: Write the v-equation of the ball and solve for the time t where $v = 0$ at the apex.

$v = v_0 + at$ where $v = 0$ m/s at apex, $v_0 = 20$ m/s, $a = -9.8$ m/s^2

$$\boxed{t = \frac{v - v_0}{a}} = \frac{(0 \text{ m/s}) - (20 \text{ m/s})}{-9.8 \text{ m/s}^2} = \boxed{2.0 \text{ s}}$$

(c) Find the largest height y_{max} reached by the ball at its apex.

Note: Write the y-equation of the ball and substitute the time t from part (b) when the ball reaches the apex.

$y = y_0 + v_0 t + \frac{1}{2}at^2$ where $y_0 = 0$ (at ground), $v_0 = 20$ m/s, $a = -9.8$ m/s^2

$\boxed{y = (20 \text{ m/s})t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2}$ where $t = 2.0$ s at apex

$$y = (20 \text{ m/s})(2.0 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(2.0 \text{ s})^2 = \boxed{20.4 \text{ m}}$$

(d) Find the time at which the ball reaches 4 m during its ascent.

Note: Write the y-equation from part (c) and solve for the time t when $y = 4$ m.

$$y = (20 \text{ m/s})t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2 = 4 \text{ m}$$

$$-(4.9 \text{ m/s}^2)t^2 + (20 \text{ m/s})t - 4 \text{ m} = 0 \quad \text{solve quadratic eqn.}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-4)}}{2(-4.9)}$$

$$t = 2.04 \pm 1.83 = \boxed{0.21 \text{ s (going up)}}$$

The larger time is for when the ball is coming down. Remember that 2.0 s is the time to reach the top.

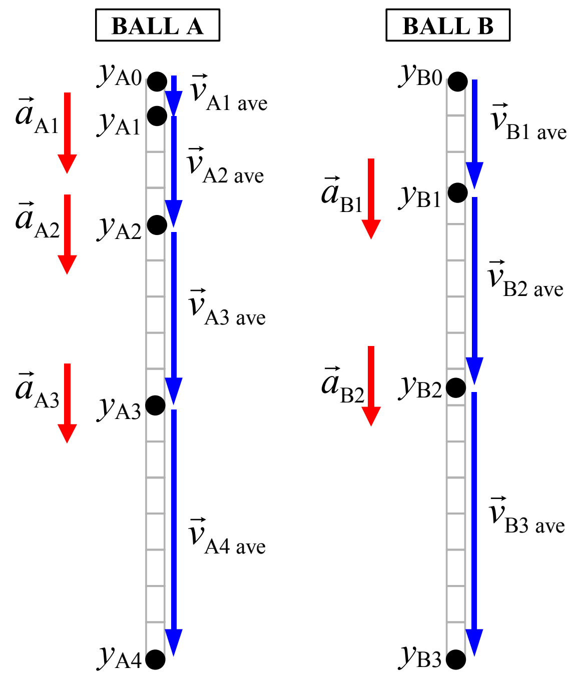
Problem #4: Vertical Motion of Two Falling Objects

You drop two water balloons off the side of a building. Balloon A is simply "let go" and takes 4 s to hit the ground. Balloon B is launched downwards using a sling shot and takes only 3 s to hit the ground.

(a) Draw the **motion diagrams** for Balloons A and B. The *y*-positions for each balloon should be marked every second and be consistent with each other. Label all average *v* vectors between the *y* -positions and the corresponding *a* vectors.

Motion Diagram

y positions separated by 1 sec intervals



(b) Find the **height *h*** of the building. Start from the *y*-equation of motion for Balloon A and assume that the **origin** is located on the **ground**. Remember to show all your work algebraically before substituting numerical values.

Note: Write the *y*-equation of Balloon A and solve for the initial height y_{A0} (or *h*). Substitute the numerical values for when the balloon reaches the ground, i.e. $y_A = 0$ m at t = 4 s.

$$y_A = y_{A0} + v_{A0}t + \tfrac{1}{2}at^2 \quad \text{where } v_{A0} = 0 \text{ m/s}, a = -g$$

$$y_A = h - \tfrac{1}{2}gt^2$$

$$\boxed{h = y_A + \tfrac{1}{2}gt^2} = 0\text{ m} + \tfrac{1}{2}(9.8\text{ m/s}^2)(4\text{ s})^2 = \boxed{78\text{ m}}$$

(c) Find the **initial velocity v_{B0}** of the "launched" Balloon B.

Note: Write the *y*-equation of Balloon B and solve for the initial velocity v_{B0}. Substitute the numerical values for when the balloon reaches the ground, i.e. $y_B = 0$ m at t = 3 s.

$$y_B = y_{B0} + v_{B0}t + \tfrac{1}{2}at^2$$

$$y_B = h + v_{B0}t - \tfrac{1}{2}gt^2$$

$$\boxed{v_{B0} = \frac{(y_B - h) + \frac{1}{2}gt^2}{t}} = \frac{(0\text{ m} - 78\text{ m}) + \frac{1}{2}(9.8\text{ m/s}^2)(3\text{ s})^2}{3\text{ s}} = \boxed{-11\text{ m/s}} \ (\sim 25\text{ mph})$$

(d) Find the **final velocity v_B** of Balloon B when it reaches the ground.

Note: Write the *v*-equation of Balloon B and substitute the numerical values for when the balloon reaches the ground, i.e. t = 3 s.

$$\boxed{v_B = v_{B0} - gt} = -11\text{ m/s} - (9.8\text{ m/s}^2)(3\text{ s}) = \boxed{-40\text{ m/s}} \ (\sim 88\text{ mph})$$