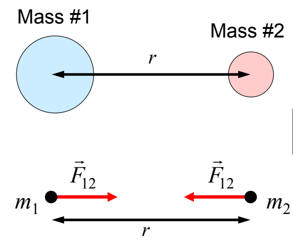
Topic 6: Gravitational Force

- Gravitational force is WEAKEST of four forces (electromagnetic, weak nuclear, strong nuclear) and is due to the mass of objects.
- Becomes important for VERY massive objects such as planets.
- Gravitational force between two objects is proportional to the product of their masses.
- Gravitational force is inversely proportional to the square of the distance between two objects.
- For calculating F_G between two objects, the entire mass of each object can be "concentrated" at its center.

Gravitational Force F_G



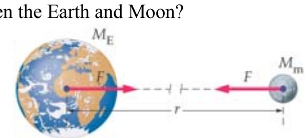
$\vec{F}_{12} = -\vec{F}_{21}$ Why?

$$\vec{F}_{12} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

Negative sign indicates an attractive force.

Gravitational Force between Earth & Moon

What is F_G between the Earth and Moon?



$$F = G\left(\frac{M_{\text{earth}}M_{\text{moon}}}{r^2}\right)$$

$$F = \frac{\left(6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}\right)\left(5.97\times10^{24}\text{ kg}\right)\left(7.35\times10^{22}\text{ kg}\right)}{(3.84\times10^{8}\text{ m})^2}$$

$$F_{Earth-Moon} = \boxed{2.0\times10^{20}\text{ N}}$$

Think fast: What is the earth-moon distance in km?

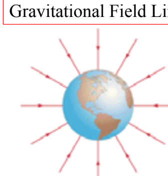
How long does light take to go from the earth to the moon?

Gravitational Field g (acceleration due to F_G)

- Mass creates a *gravitational field g* at every point in space around it.
- Field g indicates the magnitude and direction of a gravitational force on a test mass m located at position r.

$$\vec{g} = \frac{\vec{F}_G}{m} = -\frac{GM}{r^2}\hat{r}$$

Gravitational Field Lines



Density of Lines → Strength of Field
Direction of Lines → Direction of Field

Note: Like electric field $\vec{E} = \frac{\vec{F}_E}{q}$ where $\vec{F}_E$ = force, q = charge

Gravitational Field g: Value on Earth's Surface

$$F = \frac{GM_Em}{R_E^2}$$ where M_E = mass of Earth

$$\boxed{g_E = \frac{F}{m} = \frac{GM_E}{R_E^2}}$$ where g = acceleration at earth's surface ($r = R_E$)

$$\boxed{g} = \frac{\left(6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}\right)\left(5.97\times10^{24}\text{ kg}\right)}{(6.37\times10^{6}\text{ m})^2} = \boxed{9.81\text{ m/s}^2}$$

Question #6.1: Field g on Shuttle

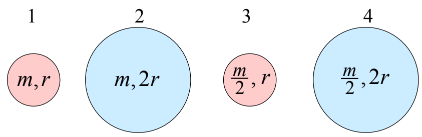
Estimate the field g_S (or acceleration) of objects inside the space shuttle as compared to the field on earth g_E. Hint: Shuttle orbits ~400 km above the earth, or at $R_S = 1.06\ R_E$.

(a) $g_S = 0$ ("weightless")
(b) $g_S = 0.9\ g_E$
(c) $g_S = 0.5\ g_E$
(d) $g_S = 0.2\ g_E$

Question #6.2: Field g on Different Planets

Five planets have relative masses and radii as shown below. The gravitational field g is smallest on the surface of which planet?

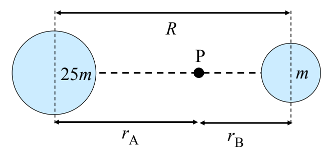
(a) 1 (b) 2 (c) 3 (d) 4



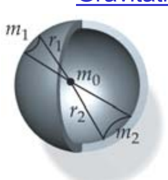
Question #6.3: Field g between Planets

Planets A and B are separated by distance R and have masses $25m$ and m. What is the value for r_A such that the gravitational field g at point P is zero?

(a) $5R/6$ (b) $25R/36$ (c) $R/25$ (d) $6R/5$



Gravitational Field g : Spherical Shell



$$\vec{g}(r<R)=0$$
$$\vec{g}(r>R)=-\frac{GM}{r^2}\hat{r}$$

Why does g equal zero inside a hollow sphere?

Note that $\dfrac{m_1}{m_2}=\dfrac{r_1^2}{r_2^2}$ since surface area $\propto r^2$.

$$\frac{g_1}{g_2}=\frac{\left(\frac{m_1}{r_1^2}\right)}{\left(\frac{m_1}{r_2^2}\right)}=\left(\frac{m_1}{m_2}\right)\left(\frac{r_2^2}{r_1^2}\right)=1 \quad \text{from above.}$$

So, g_1 and g_2 are equal in magnitude and cancel each other.

Question #6.4: Field g for a Hollow Sphere

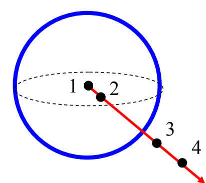
Test masses are used to measure the gravitational field g at various positions in and near a hollow spherical shell. g is smallest at point(s):

(a) 1, 2

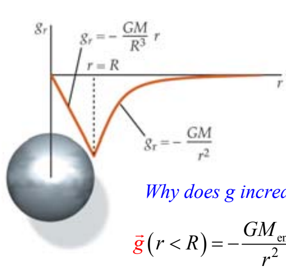
(b) 3

(c) 1, 2, 4

(d) 4



Gravitational Field g : Solid Sphere



$$\vec{g}(r<R)=-\frac{GMr}{R^3}\hat{r}$$
$$\vec{g}(r>R)=-\frac{GM}{r^2}\hat{r}$$

Why does g increase linearly inside a solid sphere?

$$\vec{g}(r<R)=-\frac{GM_{\text{encl}}}{r^2}\hat{r}=-\frac{G}{r^2}\left(\frac{Mr^3}{R^3}\right)\hat{r}$$

where $M_{\text{encl}}=M\left(\dfrac{V_{\text{encl}}}{V_{\text{tot}}}\right)=M\left(\dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right)=M\left(\dfrac{r^3}{R^3}\right)$

$$\vec{g}(r<R)=-\frac{GMr}{R^3}\hat{r}$$

Question #6.5: Field g for a Solid Sphere

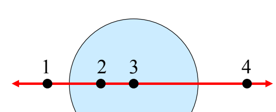
Test masses are used to measure the gravitational field g at various positions in and near a solid sphere of uniform density. g is smallest at point:

(a) 1

(b) 2

(c) 3

(d) 4

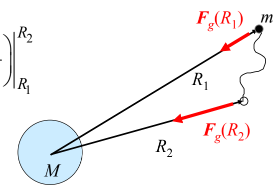


Work and Potential Energy due to Gravity

$$W_{\text{g}} = \int_{s_1}^{s_2} \vec{F}_{\text{g}} \cdot d\vec{s} = \int_{R_1}^{R_2} F_{\text{g}}\, dr \quad \text{since } \vec{F}_{\text{g}} \text{ has only a radial component}$$

$$W_{\text{g}} = \int_{R_1}^{R_2} \left(-\frac{GMm}{r^2}\right) dr = (GMm)\left(\frac{1}{r}\right)\Bigg|_{R_1}^{R_2}$$

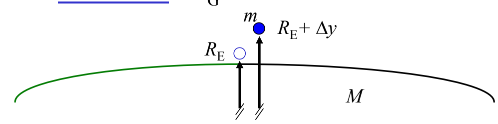
$$\boxed{W_{\text{g}} = (GMm)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)}$$



$$\Delta U_{\text{g}} = -W_{\text{g}} \quad \Rightarrow \quad \boxed{U_{\text{g}}(r) = -\frac{GMm}{r}}$$

for $U_{\text{g}} = 0$ at $r = \infty$.

Derivation: U_{G} near Earth's Surface



$$\boxed{\Delta U_{\text{g}}(R_1 \to R_2) = -(GMm)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)} = -(GMm)\left(\frac{R_1 - R_2}{R_1 R_2}\right)$$

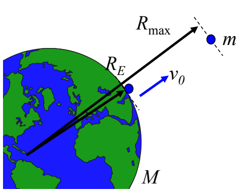
Let $R_1 = R_{\text{E}}$ and $R_2 = R_{\text{E}} + \Delta y$, where $\Delta y << R_{\text{E}}$

$$\Delta U_{\text{g}} = -(GMm)\left(\frac{R_{\text{E}} - R_{\text{E}} - \Delta y}{R_{\text{E}}(R_{\text{E}} + \Delta y)}\right)$$

$$\boxed{\Delta U_{\text{g}}} \cong -(GMm)\left(\frac{-\Delta y}{R_{\text{E}}^2}\right) = \boxed{mg\Delta y} \quad \text{where } g = \frac{GM_{\text{E}}}{R_{\text{E}}^2}$$

Energy Conversion from K to U_{G}

A projectile of mass m is launched straight up from the surface of the earth with initial speed v_0. What is the maximum distance from the center of the earth R_{max} it reaches before falling back down.



$$\boxed{\Delta K + \Delta U_{\text{g}} = 0}$$

$$\left(0 - \tfrac{1}{2}mv_0^2\right) - GMm\left(\frac{1}{R_{\max}} - \frac{1}{R_{\text{E}}}\right) = 0$$

$$v_0^2 = 2GM\left(\frac{1}{R_{\text{E}}}\right)\left(1 - \frac{R_{\text{E}}}{R_{\max}}\right)$$

$$\frac{v_0^2}{2gR_E} = 1 - \frac{R_{\text{E}}}{R_{\max}} \quad \text{where } g = \frac{GM_{\text{E}}}{R_{\text{E}}^2}$$

$$\boxed{R_{\max} = \frac{R_E}{1 - \dfrac{v_0^2}{2gR_E}}}$$

Escape Velocity

To make a projectile escape to infinity (R_{max} = infinity), the denominator in the previous equation must equal zero:

$$1 - \frac{v_0^2}{2gR_E} = 0 \quad \Rightarrow \quad \boxed{v_0 = \sqrt{2gR_E}}$$

This value of v_0 is called the "*escape velocity*" v_{esc}

	R (m)	M (kg)	g (m/s^2)	v_{esc}(m/s)
Earth	6.38×10^6	5.98×10^{24}	9.8	11,200
Moon	1.74×10^6	7.35×10^{22}	1.6	2,380
Jupiter	7.15×10^7	1.90×10^{27}	24.8	59,500
Sun	6.95×10^8	1.99×10^{30}	275	618,000

Question #6.6: Convert U_{G} to K - Part I

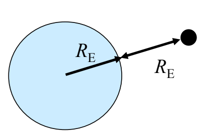
A rock is dropped from a distance R_{E} above the surface of the earth. What is its kinetic energy K when it hits the ground?

(a) $\dfrac{GMm}{R_{\text{E}}}$

(b) $\dfrac{GMm}{2R_{\text{E}}}$

(c) $\dfrac{2GMm}{R_{\text{E}}}$

(d) $\dfrac{3GMm}{2R_{\text{E}}}$



Question #6.7: Convert U_{G} to K - Part II

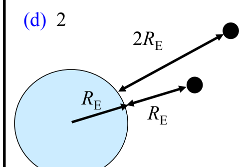
A rock is dropped from a distance R_{E} above the surface of the earth, and has kinetic energy K_1 when it hits the ground. An identical rock is dropped from twice the distance ($2R_{\text{E}}$) above the earth's surface and has kinetic energy K_2 when it hits. What is K_2 / K_1?

(a) 5/4

(b) 4/3

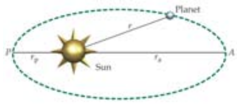
(c) 3/2

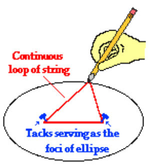
(d) 2



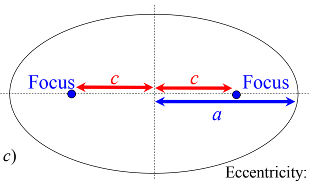
Planetary Motion: Kepler's 1st Law (shape of orbit)

All planets move in elliptical orbits with the Sun at one focus.





Loop circumference = $2(a + c)$



Eccentricity: $e = \frac{c}{a}$

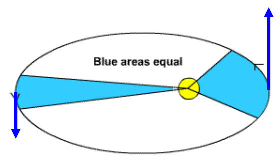
What is the definition of an ellipse?

What is the maximum value of eccentricity for an ellipse?

What is the eccentricity of a circle?

Planetary Motion: Kepler's 2nd Law (about speeds)

As a planet moves in its orbit, a line joining the planet to the sun sweeps out equal areas in equal times.



$v_{\text{perihelion}}$ "peri" = near "helios" = sun

v_{aphelion} "ap" = away from "helios" = sun

$v_{\text{perihelion}} > v_{\text{aphelion}}$

This law is a consequence of what conservation principle?

See: http://www.physics.nwu.edu/ugrad/vpl/mechanics/planets.html

Planetary Motion: Kepler's 3rd Law (about periods)

The orbital period T is related to the average orbital radius r by:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Derived by equating $F_{\text{centripetal}}$ to F_{gravity}

Calculate the period of the Earth's orbit around the Sun.

$$T^2 = \frac{4\pi^2 r^3}{GM_{Sun}} = \frac{4\pi^2 (1.5\times10^{11}\ \text{m})^3}{(6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2})(2\times10^{30}\ \text{kg})} = 1.0\times10^{15}\ \text{s}^2$$

$$T = 3.16\times10^7\ \text{s}\left(\frac{\text{day}}{3600\times24\ \text{s}}\right) = 366\ \text{days}$$

http://webphysics.davidson.edu/physlet_resources/bu_semester1/index.html

Question #6.8: Speed and Period of Mars

Mars is farther from the Sun than Earth. Which of the following statements is true?

(a) Mars has a longer period around the Sun than the Earth, but moves faster than the Earth.

(b) Mars has a longer period and moves slower than the Earth.

(c) Mars has a shorter period and moves faster than the Earth.

(d) Mars has a shorter period and moves slower than the Earth.

Question #6.9: Speed of Orbiting Planet

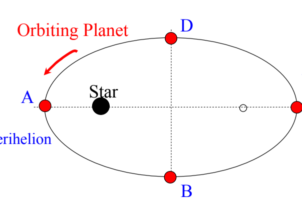
Examine the figure below of a planet orbiting a star. The planet has its fastest speed at:

(a) A

(b) B

(c) C

(d) D



v_{aphelion}

$v_{\text{perihelion}}$

Question #6.10: Radius of Orbiting Planet

The speed of an asteroid at aphelion is v_a = 10 km/s and at perihelion is v_p = 20 km/s. What is the relationship between the aphelion radius r_a to perihelion radius r_p?

(a) $r_a = \frac{1}{2} r_p$

(b) $r_a = \frac{r_p}{\sqrt{2}}$

(c) $r_a = \sqrt{2}\, r_p$

(d) $r_a = 2 r_p$