Topic #4: Impulse and Momentum

Scalar

WORK: $W_{\text{net}} = \int_{s_i}^{s_f} \vec{F}_{\text{net}} \cdot d\vec{s} = \Delta K$ where $K = \frac{1}{2}mv^2$

Simplest Definition of Work (for constant force):
Work equals force times parallel distance.

Net work causes object to speed up or slow down.

Vector

IMPULSE: $\vec{I} = \int_{t_i}^{t_f} \vec{F}_{\text{net}}\, dt = \Delta\vec{p}$ where $\vec{p} = m\vec{v}$

Impulse equals force times amount of time.

Net impulse causes object to speed up or slow down and/or turn.

Question #4.1: Momentum and Kinetic Energy #1

If a net force is exerted on an object for a given amount of time, then

(a) **momentum** may change and **kinetic energy** always changes.

(b) **momentum** may change and **kinetic energy** may change.

(c) **momentum** always changes and **kinetic energy** always changes.

(d) **momentum** always changes **kinetic energy** may change.

Question #4.2: Momentum and Kinetic Energy #2

If a net force is exerted on an object over a distance Δx, then

(a) **momentum** may change and **kinetic energy** always changes.

(b) **momentum** may change and **kinetic energy** may change.

(c) **momentum** always changes and **kinetic energy** always changes.

(d) **momentum** always changes **kinetic energy** may change.

Question #4.3: Two Pushed Boxes #1

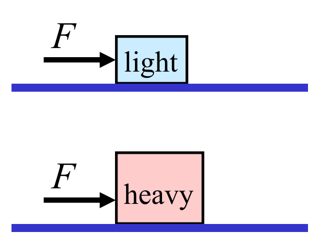
A lighter and heavier box are at rest on a frictionless surface.
The same force F pushes on each box for 1 second.
Which box has the larger final MOMENTUM after the force acts?

(a) lighter

(b) same

(c) heavier

(d) insufficient information



Question #4.4: Two Pushed Boxes #2

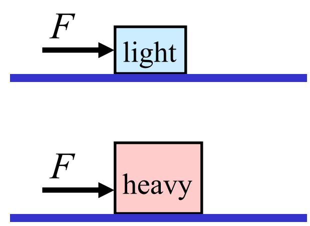
A lighter and heavier box are at rest on a frictionless surface.
The same force *F* pushes on each box for 1 second.
Which box has the larger final KINETIC ENERGY after the force acts?

(a) lighter

(b) same

(c) heavier

(d) insufficient information



Question #4.5: Two Pushed Boxes #3

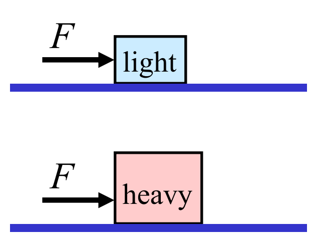
A lighter and heavier box are at rest on a frictionless surface.
The same force *F* pushes on each box for a distance of 1 meter.
Which box has the larger final KINETIC ENERGY after the force acts?

(a) lighter

(b) same

(c) heavier

(d) insufficient information



Question #4.6: Two Pushed Boxes #4

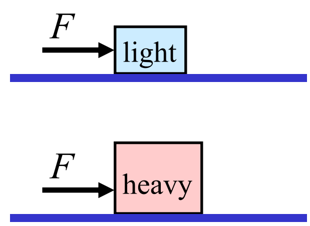
A lighter and heavier box are at rest on a frictionless surface.
The same force F pushes on each box for a distance of 1 meter.
Which box has the larger final MOMENTUM after the force acts?

(a) lighter

(b) same

(c) heavier

(d) insufficient information



Question #4.7: Change in Momentum

The cart's change in momentum is:

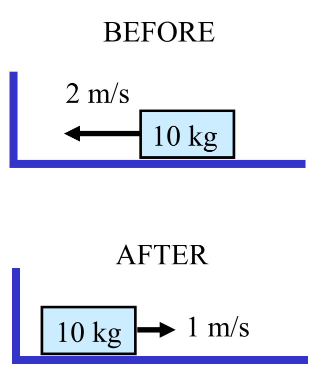
(a) -30 kg m/s

(b) -20 kg m/s

(c) 10 kg m/s

(d) 20 kg m/s

(e) 30 kg m/s



Question #4.8: Two Falling Balls #1

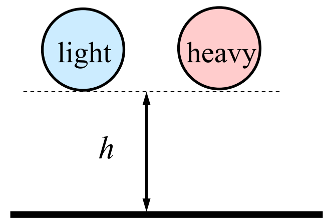
A lighter and a heavier ball are both dropped from the same height h at the same time. Which ball has a larger final MOMENTUM when it hits the ground?

(a) lighter

(b) same

(c) heavier

(d) insufficient info



Question #4.9: Two Falling Balls #2

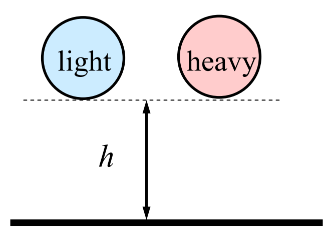
A lighter and a heavier ball are both dropped from the same height h at the same time. Which ball has a larger final KINETIC ENERGY when it hits the ground?

(a) lighter

(b) same

(c) heavier

(d) insufficient info



Question #4.10: Two Falling Balls #3

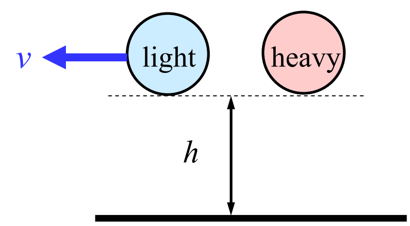
A lighter and a heavier ball are both dropped from the same height h at the same time. The lighter ball has an initial horizontal velocity. Which ball has a larger final MOMENTUM when it hits the ground?

(a) lighter

(b) same

(c) heavier

(d) insufficient info



Momentum and Force

Single particle system

$$\vec{p} = m\vec{v}$$

$$\vec{F}_{\text{net}} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

Multi-particle System CM = Center of Mass

$$\vec{p}_{\text{sys}} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i\vec{v}_i = M\vec{v}_{CM} \qquad \text{where } v_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{v}_i$$

$$\vec{F}_{\text{net}} = \sum_i m_i\vec{a}_i = M\vec{a}_{CM} = M\frac{d\vec{v}_{CM}}{dt} = \frac{d\vec{p}_{\text{sys}}}{dt}$$

Only a net EXTERNAL force can change the momentum of a system!

Velocity and Acceleration of SYSTEM of particles

The velocity and acceleration of a system is the <u>WEIGHTED AVERAGE</u> of the velocity and acceleration of all the particles in the system >>> CENTER OF MASS variables.

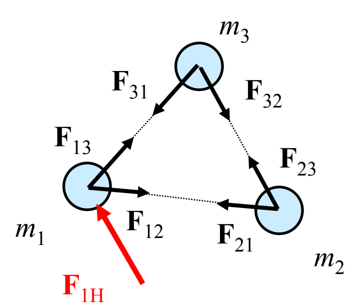
$$\boxed{\vec{r}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i} \qquad \text{where} \quad M = \sum_{i=1}^{N} m_i$$

$$\boxed{\vec{v}_{CM}} = \frac{d\vec{r}_{CM}}{dt} = \frac{1}{M}\sum_{i=1}^{N} m_i \frac{d\vec{r}_i}{dt} = \boxed{\frac{1}{M}\sum_{i=1}^{N} m_i \vec{v}_i}$$

$$\boxed{\vec{a}_{CM}} = \frac{d\vec{v}_{CM}}{dt} = \frac{1}{M}\sum_{i=1}^{N} m_i \frac{d\vec{v}_i}{dt} = \boxed{\frac{1}{M}\sum_{i=1}^{N} m_i \vec{a}_i}$$

Newton's 2nd Law for System of Particles

- Given a system of three particles, where each particle interacts with every other, and an <u>external force</u> pushes on <u>particle 1</u>.
- All of the "<u>internal</u>" forces <u>cancel</u> !! Only the "<u>external</u>" force <u>matters</u> !!



$$\sum_i \vec{F}_i = \left(\vec{F}_{13} + \vec{F}_{12} + \vec{F}_{1H}\right) + \left(\vec{F}_{21} + \vec{F}_{23}\right) + \left(\vec{F}_{31} + \vec{F}_{32}\right)$$

$$\boxed{\sum_i \vec{F}_i = \vec{F}_{1H}} \qquad \text{All 3rd law pairs cancel!}$$

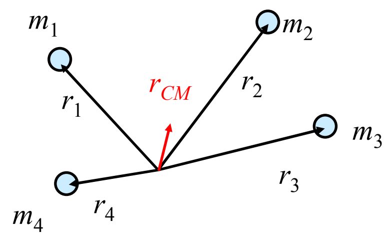
In general, $$\boxed{\frac{d\vec{p}_{sys}}{dt} = \sum_i \vec{F}_i = \vec{F}_{\text{Net, Ext}} = M\vec{a}_{CM}}$$

What is the Center of Mass for a System?

How do we describe the "position" of a system made up of many parts?

Define the Center of Mass r_{CM} (ave. position) using a WEIGHTED AVERAGE.

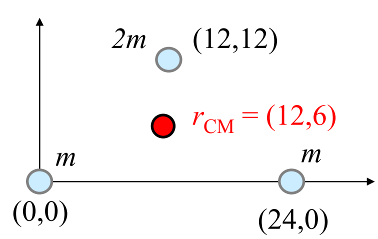
$$\vec{r}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i}$$



Cartesian Form: $(x_{CM}, y_{CM}, z_{CM}) = \left(\frac{\sum_i m_i x_i}{M}, \frac{\sum_i m_i y_i}{M}, \frac{\sum_i m_i z_i}{M} \right)$

Problem #4.1: Center of Mass for 3 points

Find the center of mass for the three point masses given below.



$$\boxed{x_{CM}} = \frac{\sum_i m_i x_i}{M} = \frac{m(0) + (2m)(12) + m(24)}{4m} = \boxed{12}$$

$$\boxed{y_{CM}} = \frac{\sum_i m_i y_i}{M} = \frac{m(0) + (2m)(12) + m(0)}{4m} = \boxed{6}$$

Question #4.11: Center of Mass for Multiple Objects

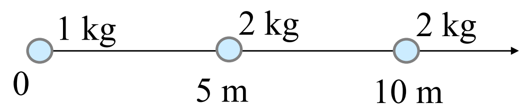
What is the center of mass for the three point masses given below?

(a) 5 m

(b) 6 m

(c) 7 m

(d) 8 m

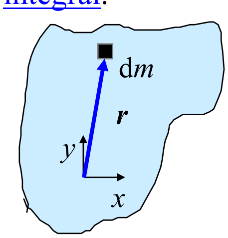


Center of Mass: Continuous Mass Distributions

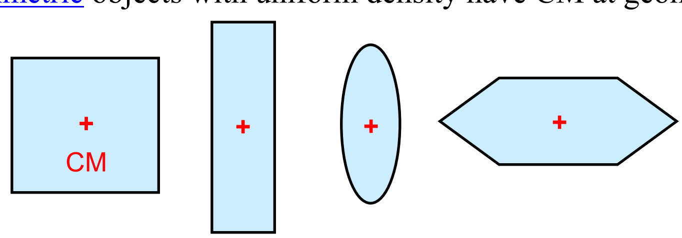
For a continuous solid, CM requires an integral.

$$\boxed{\vec{r}_{CM}} = \frac{\int \vec{r}\, dm}{\int dm} = \frac{\int \vec{r}\, dm}{M}$$

(independent of choice for origin)

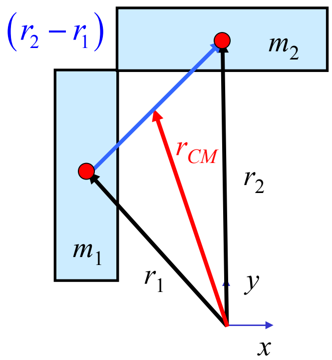


Symmetric objects with uniform density have CM at geometrical center.



Center of Mass: Two Symmetric Distributions

CM for a combination of symmetric objects is the “point-like” combination of the CM for each symmetric object.



$$\vec{r}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \vec{r}_1 + \frac{m_2}{M}(\vec{r}_2 - \vec{r}_1)$$

(same as for point masses)

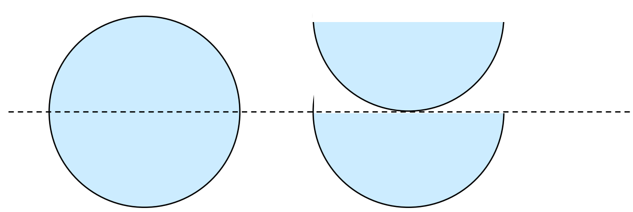
Question #4.12: Center of Mass

If the disk shown in (1) is cut in half and the pieces arranged as shown in (2), then where is the CM of (2) as compared to (1)?

(a) higher

(b) lower

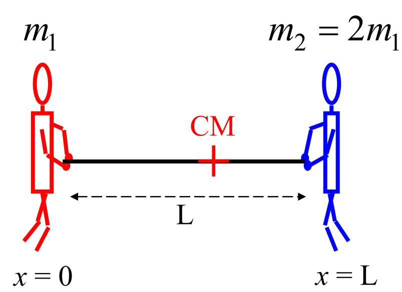
(c) same



(1) (2)

Problem #4.2: Center of Mass for Internal Motion

Two astronauts are at rest in outer space and are connected by a light rope. They begin to pull towards each other. Where do they meet?



They start at rest, so $v_{CM} = 0$.

v_{CM} remains zero because there are no external forces.

So, the CM does not move and they will meet at the CM.

$$\boxed{x_{cm}} = \frac{\sum_i m_i x_i}{M} = \frac{m_1(0) + 2m_1(L)}{m_1 + 2m_1} = \frac{2m_1(L)}{3m_1} = \boxed{\frac{2L}{3}}$$

Question #4.13: Center of Mass for Internal Motion

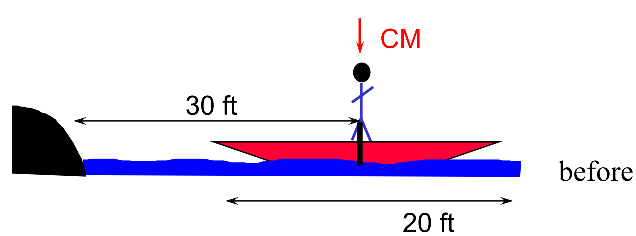
A man weighs the same as his 20-ft canoe. First, he stands in the center of the motionless canoe at a distance 30 ft from shore. Next, he walks left 10 ft toward the shore to the end of the canoe. What is his new distance from the shore? (Hint: CM of man+canoe does not move since $F_{\text{net}} = 0$.)

(a) 15 ft

(b) 20 ft

(c) 25 ft

(d) 30 ft



Conservation of Momentum

$$\boxed{\vec{F}_{\text{net}} = \frac{d\vec{p}_{\text{sys}}}{dt}}$$

$$\text{For } \vec{F}_{\text{net}} = 0, \quad \frac{d\vec{p}_{\text{sys}}}{dt} = 0 \quad \text{or} \quad \boxed{\vec{p}_{\text{i}} = \vec{p}_{\text{f}}}$$

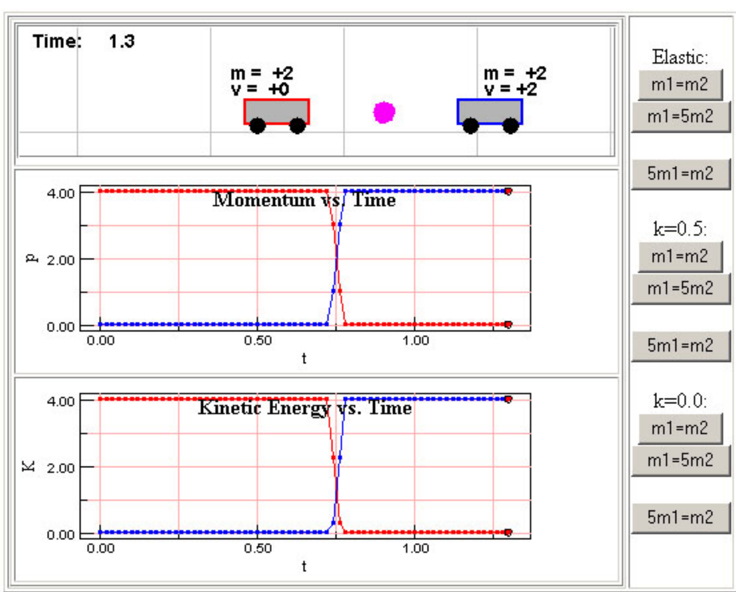
$$\boxed{m_1\vec{v}_{1\text{i}} + m_2\vec{v}_{2\text{i}} + \text{ etc.} = m_1\vec{v}_{1\text{f}} + m_2\vec{v}_{2\text{f}} + \text{ etc.}}$$

Momentum conservation for NO net external forces.

- Momentum conservation is a fundamental principle in physics.
- Applies to any direction in which no net force is applied, i.e. x-y-z directions are *independent* of each other.

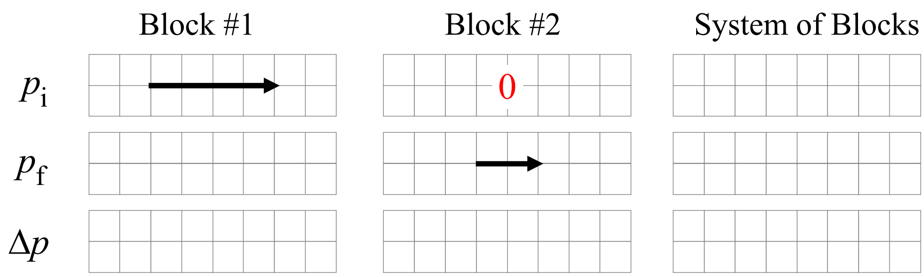
Conservation of Momentum: Collisions

http://webphysics.davidson.edu/physlet_resources/bu_semester1/index.html Collisions



Problem #4.3: Conservation of Momentum

Block #1 collides with a stationary block #2 and no net forces act on the system of blocks. Draw the momentum vectors indicated below with correct magnitude.



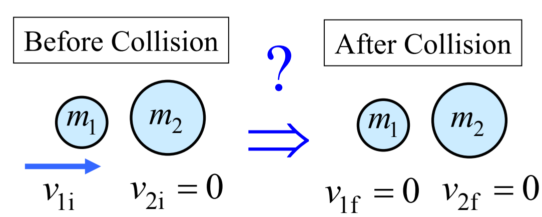
Question #4.14: Momentum of Two-body Collision

A ball collides with another stationary ball. Is it possible for both balls to be at rest after the collision? (assume no external forces)

(a) yes

(b) no

(c) depends on v_{1i}



Question #4.15: Perfectly Inelastic Collision

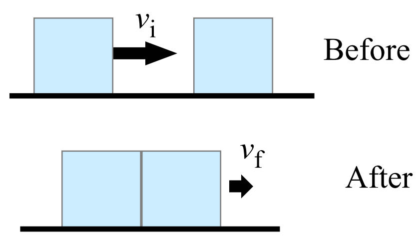
A box slides on a frictionless surface with initial velocity v_i . It then collides with a stationary identical box and the boxes stick together. What is final velocity v_f of the two "stuck" boxes?

(a) $v_f = v_i$

(b) $v_f = \frac{1}{\sqrt{2}} v_i$

(c) $v_f = \frac{1}{4} v_i$

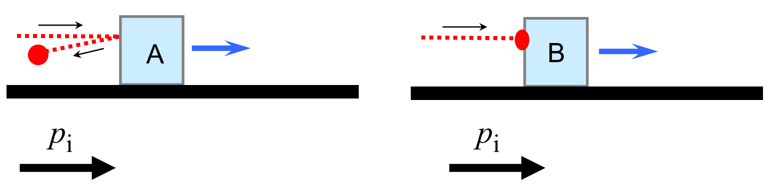
(d) $v_f = \frac{1}{2} v_i$



Question #4.16: Two Types of Collisions

Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface. The ball hitting box A bounces back, while the ball hitting box B gets stuck. Which box moves faster after the collision?

(a) Box A (b) Box B (c) same

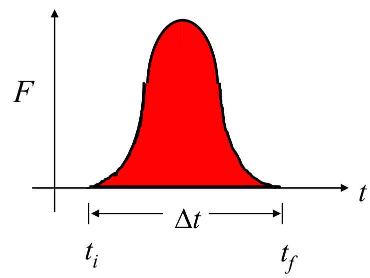


Impulse and Average Force

- Impulse I is the area under the force-vs.-time curve. It equals the change of momentum during that time period.
- The average force during the same time period is the total impulse divided by the time period.

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}_{net}\, dt = \int_{t_i}^{t_f} \left(\frac{d\vec{p}}{dt}\right) dt = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

$$\vec{F}_{ave} = \frac{\vec{I}}{\Delta t}$$



Problem #4.4: Impulse during Car Crash

If a car and 80-kg dummy drive into a wall at 25 m/s, estimate the average force that the seatbelt exerts on the dummy upon impact. Assume the car moves 1 m after impact as a result of its crumple zone.

$$\left|\vec{I}\right| = \Delta\vec{p} = m\left|v_f - v_i\right| = (80\text{ kg})(25\text{ m/s}) = 2000\text{ Ns}$$

$$\Delta t = \frac{\Delta x}{v_{ave}} = \frac{\Delta x}{\frac{1}{2}(v_f + v_i)} = \frac{1\text{ m}}{\frac{1}{2}(0 + 25\text{ m/s})} = 0.08\text{ s}$$

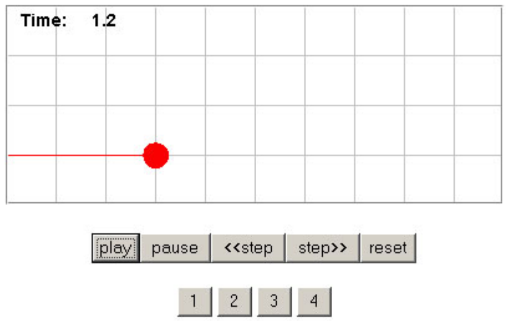
$$\vec{F}_{ave} = \frac{\vec{I}}{\Delta t} = \frac{2000\text{ Ns}}{0.08\text{ s}} = 25{,}000\text{ N} \quad \text{and} \quad a_{ave} = \frac{F_{ave}}{m} \approx 300\text{ m/s}^2$$

How many g's does the dummy experience during this crash?

Question #4.17: Motion due to Impulse

http://webphysics.davidson.edu/physlet_resources/bu_semester1/index.html Impulse

A mass moving in the x direction at constant speed on a frictionless table is subjected to a force in the y-direction. The force is applied for 0.5 seconds starting at t=2 seconds. Which of the following situations corresponds to the motion of the mass?

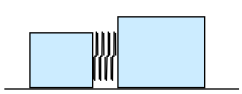


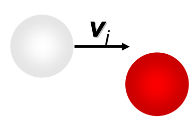
(a) Path 1 (b) Path 2 (c) Path 3 (d) Path 4

Elastic and Inelastic Collisions

An *elastic* collision occurs when kinetic energy is CONSERVED.

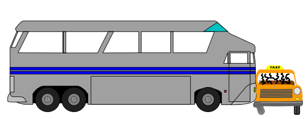
e.g., carts colliding with a spring between them, billiard balls, etc.





An *inelastic* collision occurs when kinetic energy is NOT CONSERVED.

A perfectly *inelastic* collision occurs when both objects STICK together and have the same final velocity.

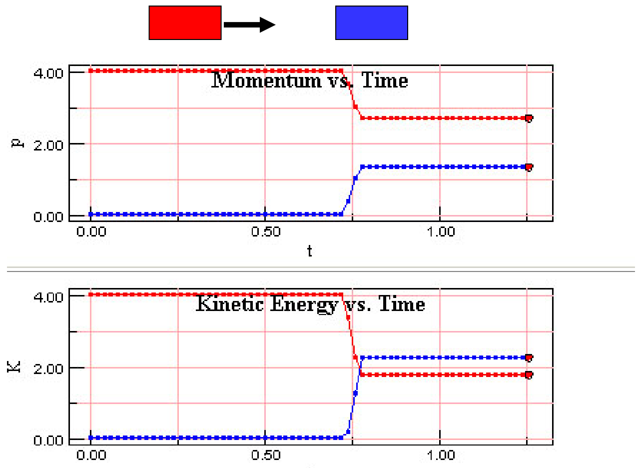


http://webphysics.davidson.edu/physlet_resources/bu_semester1/index.html Collisions

Question #4.18: Type of Collision I

Momentum

Kinetic Energy

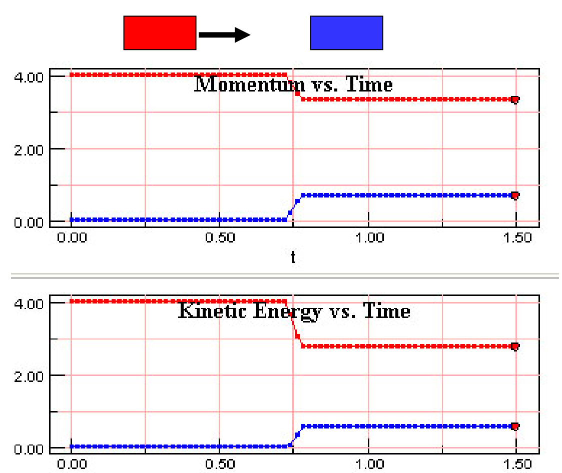


(a) Inelastic, Red heavier
(b) Inelastic, Red lighter
(c) Elastic, Red heavier
(d) Elastic, Red lighter

Question #4.19: Type of Collision II

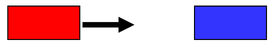
Momentum

Kinetic Energy

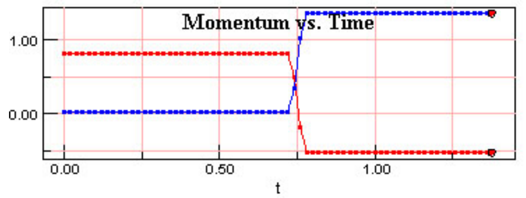


(a) Inelastic, Red heavier
(b) Inelastic, Red lighter
(c) Elastic, Red heavier
(d) Elastic, Red lighter

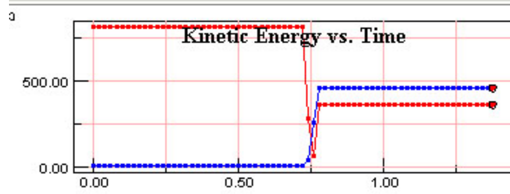
Question #4.20: Type of Collision III



Momentum

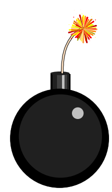


Kinetic Energy



(a) Inelastic, Red heavier
(b) Inelastic, Red lighter
(c) Elastic, Red heavier
(d) Elastic, Red lighter

Inelastic "Explosion" (or collision)

Before the explosion:  $m_1 + m_2$

After the explosion:

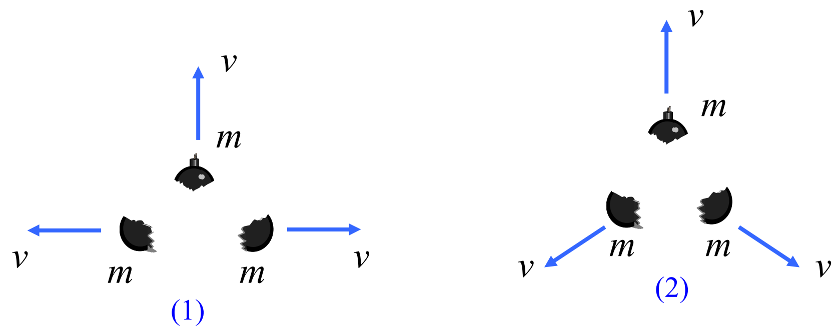
v_{1f} m_1 m_2 v_{2f}

$$p = 0 = m_1 v_{1f} + m_2 v_{2f} \quad \Rightarrow \quad \boxed{m_1 v_{1f} = -m_2 v_{2f}}$$

Question #4.21: Inelastic Explosions

A bomb explodes into 3 identical pieces. Which of the following configurations of velocities is possible?

(a) 1 (b) 2 (c) both (d) insufficient info.



Problem #4.5: Perfectly Inelastic Collision

A block of mass m_2 is initially at rest on a frictionless horizontal surface. A bullet of mass m_1 is fired at the block with an initial velocity v_{1i}. If the bullet lodges in the block, find the final speed v_f of the block and bullet and its final kinetic energy (in terms of initial K_i).

$$\boxed{p_i = p_f}$$

$$m_1 v_{1i} = (m_1 + m_2) v_f$$

$$\boxed{v_f} = \frac{m_1}{(m_1 + m_2)} v_{1i}$$

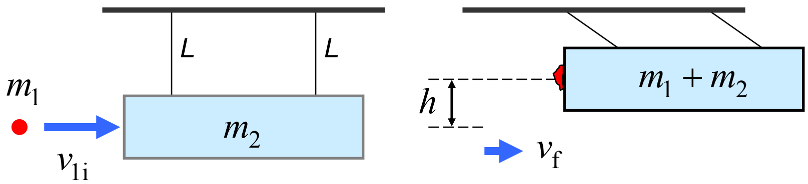
$$K_f = \tfrac{1}{2}(m_1 + m_2) v_f^2 = \frac{\tfrac{1}{2}(m_1 + m_2) m_1^2}{(m_1 + m_2)^2} v_{1i}^2$$

$$\boxed{K_f} = \frac{m_1}{(m_1 + m_2)} \left(\tfrac{1}{2} m_1 v_{1i}^2\right) = \boxed{\frac{m_1}{(m_1 + m_2)} K_i}$$

Under what circumstances could the final K_f approach the initial K_i?

Problem #4.6: Perfectly Inelastic Ballistic Pendulum

A projectile of mass m_1 moves horizontally with speed v_1 and strikes a stationary, suspended mass m_2. The combined masses rise to a height of h. What is the initial speed v_{1i} of the projectile?



Use momentum conservation in x-direction for collision:

$$m_1 v_{1i} = (m_1 + m_2) v_f \quad \Rightarrow \boxed{v_{1i}} = \left(1 + \frac{m_2}{m_1}\right) v_f = \boxed{\left(1 + \frac{m_2}{m_1}\right)\sqrt{2gh}}$$

where v_f is given by energy conservation AFTER collision.

$$\tfrac{1}{2}(m_1 + m_2) v_f^2 = (m_1 + m_2) gh \quad \Rightarrow \boxed{v_f = \sqrt{2gh}}$$

Question #4.22: Perfectly Inelastic Collision

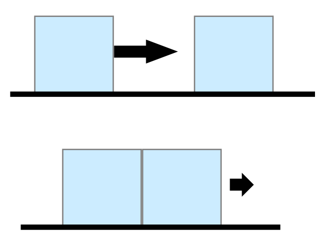
A box sliding on a frictionless surface collides and sticks to a second identical box which is initially at rest. What is the final kinetic energy K_f of the stuck boxes?

(a) $E_f = E_i$

(b) $E_f = \frac{1}{\sqrt{2}} E_i$

(c) $E_f = \frac{1}{4} E_i$

(d) $E_f = \frac{1}{2} E_i$

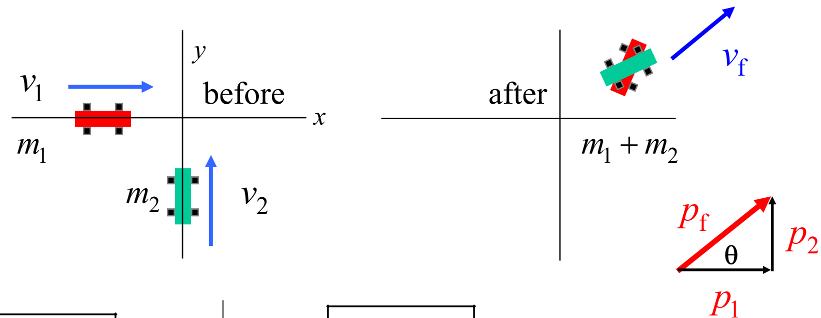


Before

After

Problem #4.7: Perfectly Inelastic Collision in 2D

If car 1 heads east and crashes into car 2 heading north, what is their resultant direction if the cars "stick" together and never use their brakes?



$$\boxed{p_{xi} = p_{xf}}$$

$$m_1 v_1 = (m_1 + m_2) v_{xf}$$

$$\boxed{v_{xf}} = \frac{m_1}{(m_1 + m_2)} v_1$$

$$\boxed{p_{yi} = p_{yf}}$$

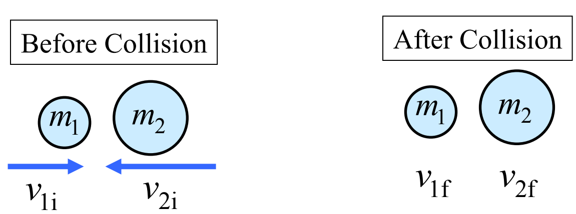
$$m_2 v_2 = (m_1 + m_2) v_{yf}$$

$$\boxed{v_{yf}} = \frac{m_2}{(m_1 + m_2)} v_2$$

$$\boxed{\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)}$$

Problem #4.8a: Elastic Collision

Two blocks with masses m_1 and m_2 and initial velocities v_{1i} and v_{2i} collide elastically with each other. How do their relative velocities change before and after the collision?

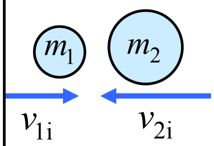


$$\boxed{p_i = p_f} \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

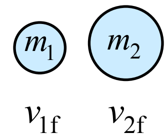
$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i}) \quad \text{Eqn \#1}$$

Problem #4.8b: Elastic Collision

Before



After



Kinetic Energy is CONSERVED in ELASTIC collisions.

$$\boxed{K_i = K_f} \Rightarrow \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

$$m_1 \left(v_{1i}^2 - v_{1f}^2\right) = m_2 \left(v_{2f}^2 - v_{2i}^2\right)$$

$$m_1 \left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) = m_2 \left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right) \text{ Eqn \#2}$$

Divide Eqn #2 by Eqn #1: $m_1 \left(v_{1i} - v_{1f}\right) = m_2 \left(v_{2f} - v_{2i}\right)$

$$\left(v_{1i} + v_{1f}\right) = \left(v_{2f} + v_{2i}\right) \quad \text{or} \quad \boxed{v_{2f} - v_{1f} = -\left(v_{2i} - v_{1i}\right)}$$

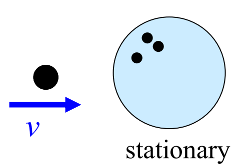
The speed of recession equals the speed of approach.

Question #4.23: Elastic Collision

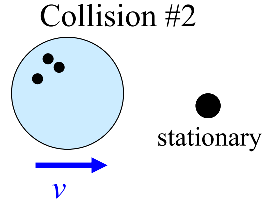
Two elastic collisions are shown. In collision #1, a golf ball with speed v hits a stationary bowling ball head on. In collision #2, a bowling ball with the same speed v hits a stationary golf ball. In which collision does the golf ball have the greater speed after the collision?

(a) 1 (b) 2 (c) same (d) insufficient info.

Collision #1



Collision #2

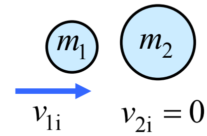


Problem #4.9: Elastic Collision - One Mass at Rest

For m_2 initially at rest $(v_{2i} = 0)$, relate final velocities to v_{1i}.

Momentum Conservation: $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$ Eqn #1



Energy Conservation: $v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$

$v_{2f} - v_{1f} = v_{1i}$ for $v_{2i} = 0$ Eqn #2

Combine Eqns #1 and #2 to solve for v_{1f} and v_{2f}

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

What happens for collisions where $m_1 = m_2$, $m_1 >> m_2$, and $m_1 << m_2$?

Question #4.24: Elastic Collision

A small rubber ball is placed on top of a big basketball. They are dropped and have speed v just before hitting the ground. What is the speed of the rubber ball after the balls hit the ground and come back up?

(Assume that the collision is elastic and that the basketball does not change speed, i.e. it has a very large mass compared to the rubber ball.)

(a) v (b) $2v$ (c) $3v$ (d) $4v$