Topic #1 (cont.): Motion - Part 2

- Vectors:
  Magnitude, direction, cartesian components,
  vector addition/subtraction, 3D motion equations

- Projectile Motion - position equations, flight time, range

$$v_x = v_{0x}$$
$$v_y = v_{0y} - gt$$

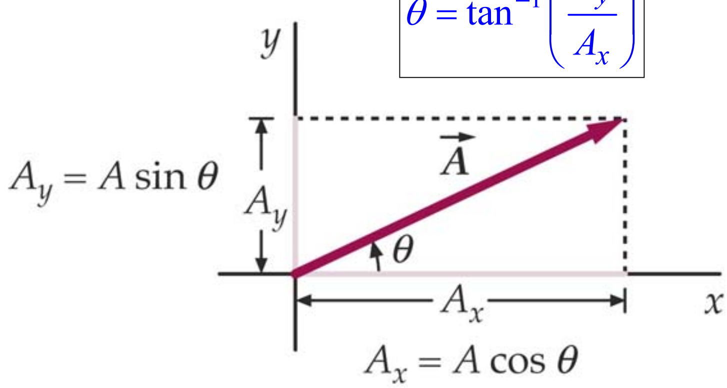
$$x = v_{0x} t$$
$$y = y_0 + v_{0y} t - \tfrac{1}{2} g t^2$$

- Circular Motion - centripetal acceleration

$$a_c \text{ or } a_\perp = \frac{v^2}{r}$$

Vector Definitions: Magnitude, Angle, Components

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

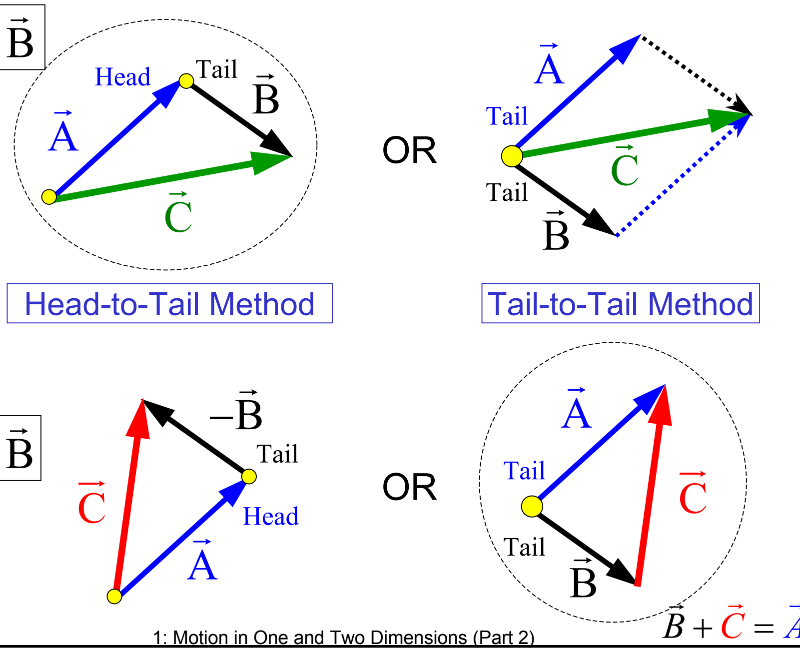


Magnitude $\left|\vec{A}\right| = \sqrt{A_x^{\,2} + A_y^{\,2}}$

Graphical Vector Addition/Subtraction

$\vec{C} = \vec{A} + \vec{B}$

$\vec{C} = \vec{A} - \vec{B}$

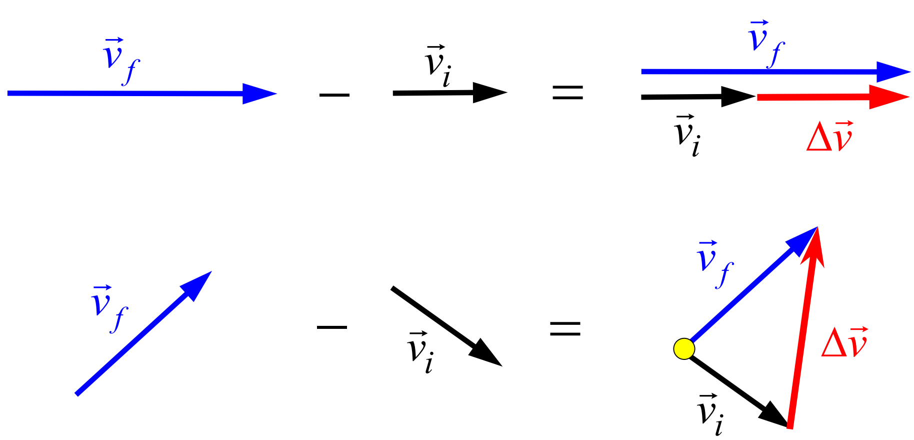


Graphical Subtraction: Velocity Vectors

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

OR $\vec{v}_f = \vec{v}_i + \Delta\vec{v}$

What vector Δv is added to the initial velocity v_i to give the final velocity v_f?



Question #1.14: Graphical Vector Subtraction

Given the initial and final velocity vectors below, what is the difference vector $\Delta v = v_f - v_i$?

(a) $\Delta\vec{v} =$ 

(b) $\Delta\vec{v} =$ 

(c) $\Delta\vec{v} =$ 

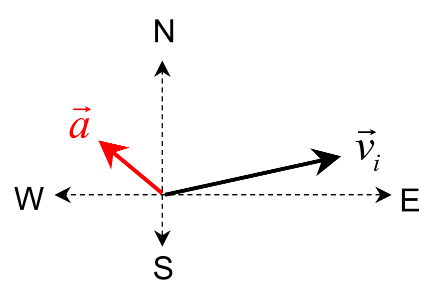
(d) $\Delta\vec{v} =$ 

(e) None of the above.

Question #1.15: Velocity and Acceleration Vectors

Given the instantaneous initial velocity and acceleration vectors below, what is the object doing?

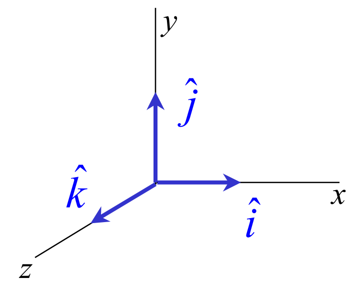
(a) Speeding up & turning South
(b) Speeding up & turning North
(c) Slowing down & turning South
(d) Slowing down & turning North



Cartesian Unit Vectors

Unit vectors have no units and a length of “one”.

They are only used to specify direction.

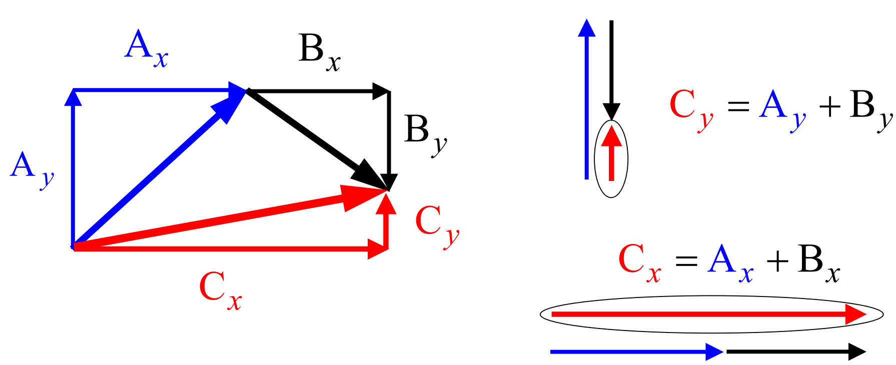


$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Component Vector Addition

$$\vec{C} = \vec{A} + \vec{B} = \left(A_x\hat{i} + A_y\hat{j}\right) + \left(B_x\hat{i} + B_y\hat{j}\right)$$

$$\vec{C} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$



Question #1.16: Summation of Vectors

What is the resultant vector D from adding **A**+**B**+**C**?

$$\vec{A} = 2\,\hat{j} + \hat{k}$$
$$\vec{B} = 3\,\hat{i} + 2\,\hat{k}$$
$$\vec{C} = \hat{i} - 4\,\hat{j} + 2\hat{k}$$

(a) $\vec{D} = 3\,\hat{i} + 5\,\hat{j} + 5\hat{k}$

(b) $\vec{D} = 5\,\hat{i} - 2\,\hat{j} + 5\hat{k}$

(c) $\vec{D} = 4\,\hat{i} - 2\,\hat{j} + 5\hat{k}$

(d) $\vec{D} = 6\,\hat{i} - 4\,\hat{j} + 5\hat{k}$

Definitions of *x, v, a* in 3-D

Vector Equations

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

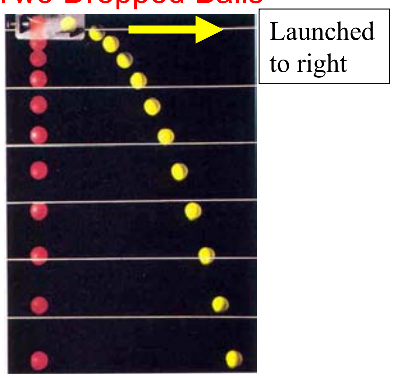
$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$$
$$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$
$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

Scalar *x*-component Eqns.

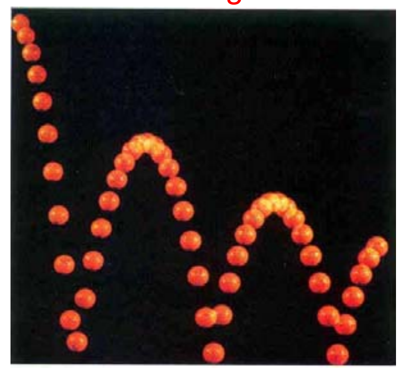
$$a_x = a_x$$
$$v_x = v_{0x} + a_x t$$
$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

Projectile Motion: Vertical & Horizontal Motion

Two Dropped Balls



Bouncing Ball

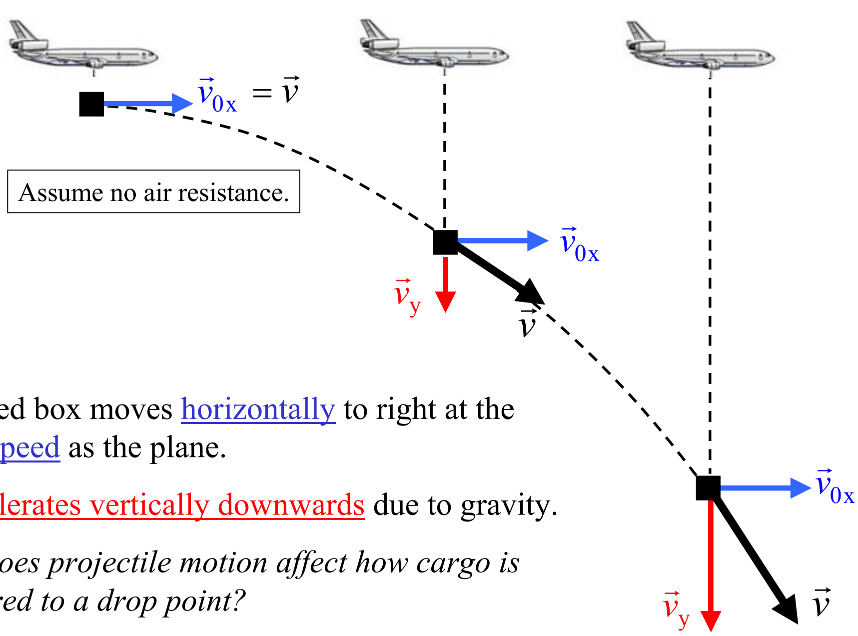


Time-lapse photos taken at equal time intervals.

- Notice that both dropped balls (straight down and to side) fall at the same rate.
- The horizontal and vertical motions of the balls are INDEPENDENT of each other.

Figure from Halliday, Resnick, & Walker

Projectile Motion: Dropped Box from Plane

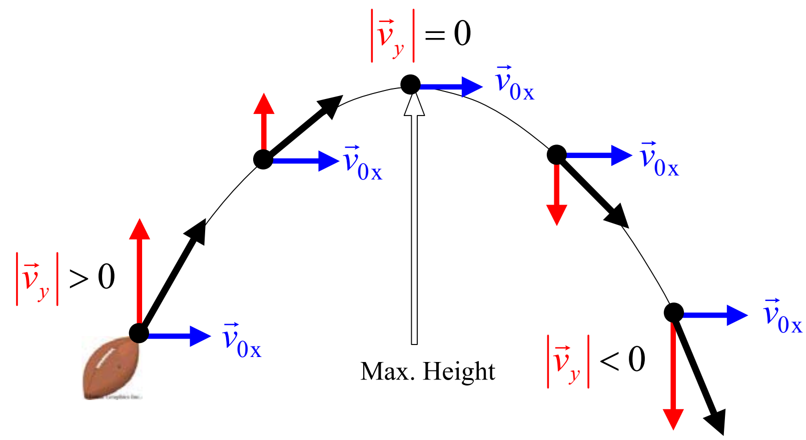


Dropped box moves horizontally to right at the same speed as the plane.

It accelerates vertically downwards due to gravity.

How does projectile motion affect how cargo is delivered to a drop point?

Projectile Motion: Kicked Ball



- Kicked ball travels at a constant horizontal velocity (depends on kick angle) and changing vertical velocity (due to gravity).
- *How could you increase the maximum height of the football? How about its range?*

Question #1.17: Parabolic Paths of Two Balls #1

Two metal balls of mass m_1 and m_2, where $m_2 = 2\ m_1$, roll off a table at the same height with the same speed. The balls hit the floor at horizontal distances of x_1 and x_2 from the base of the table. Which statement is true?

(a) x_2 (heavier ball) << x_1 (lighter ball)

(b) $x_2 = 0.5\ x_1$

(c) $x_2 = x_1$

(d) $x_2 = 2\ x_2$

(e) Insufficient information.

Question #1.18: Parabolic Paths of Two Balls #1

Two metal balls of equal mass roll off tables of different heights h_1 and h_2, where $h_2 = 2\, h_1$. The balls have the same initial speed and hit the floor at horizontal distances of x_1 and x_2 from the base of the table. Which statement is true?

(a) x_2 (higher ball) $= 2\, x_1$ (lower ball)

(b) $x_2 = x_1$

(c) $x_2 = 0.5\, x_1$

(d) Insufficient information

(e) None of the above

Question #1.19: Projectile Speed

A projectile is fired at 35° above the horizontal. At the highest point in its trajectory, its speed is 200 m/s. The projectile's initial velocity had a horizontal component of:

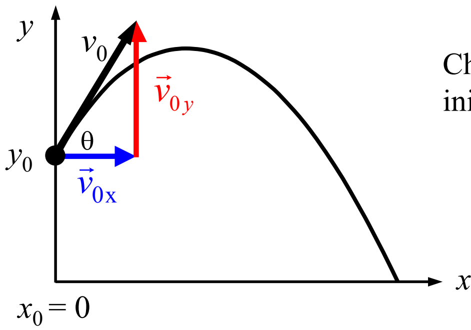
(a) 0 m/s

(b) (200 m/s) cos 35°

(c) (200 m/s) sin 35°

(d) 200 m/s

Projectile Motion: General Formulas for x, y, etc.



Choose origin on ground below initial location of projectile.

$$\boxed{\begin{aligned} v_x &= v_{0x} \\ v_y &= v_{0y} - gt \end{aligned}}$$

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

where $x_0 = 0,\ a_x = 0$

$$\boxed{x = v_{0x}t}$$

$$x = (v_0 \cos\theta)t$$

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

where $a_y = -g$

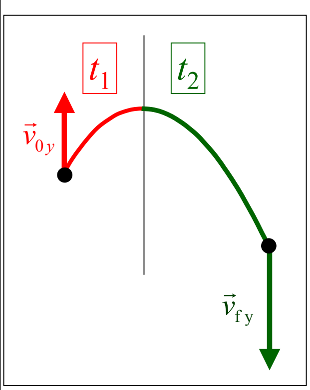
$$\boxed{y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2}$$

$$\Delta y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$

Projectile Motion: General Formula for Flight Time

$\underline{y\text{-eqn}}$: $\left(\tfrac{1}{2}g\right)t^2 + \left(-v_{0y}\right)t + \Delta y = 0$ where $\Delta y = y - y_0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{v_{0y} \pm \sqrt{\left(v_{0y}\right)^2 - 4\left(\tfrac{1}{2}g\right)(\Delta y)}}{2\left(\tfrac{1}{2}g\right)}$$



$$\boxed{t = \frac{v_{0y}}{g} \pm \frac{\sqrt{\left(v_{0y}\right)^2 - 2g\Delta y}}{g}}$$

No v_{0x} dependence!

Remember "Energy" Eqn. $v_f{}^2 = v_0{}^2 - 2g\Delta y$

$$\boxed{t = \frac{v_{0y}}{g} \pm \frac{v_{fy}}{g} = t_1 \pm t_2} \qquad \boxed{t_{\text{full arc}} = \frac{2v_{0y}}{g}}$$

where t_1 = time from initial position to apex
t_2 = time from final position to apex

Question #1.20: Flight Times of Different Parabolas

Does Ball B spend more/ the same/ less time in the air than Ball A?

(a) More time (b) Same time (c) Less time

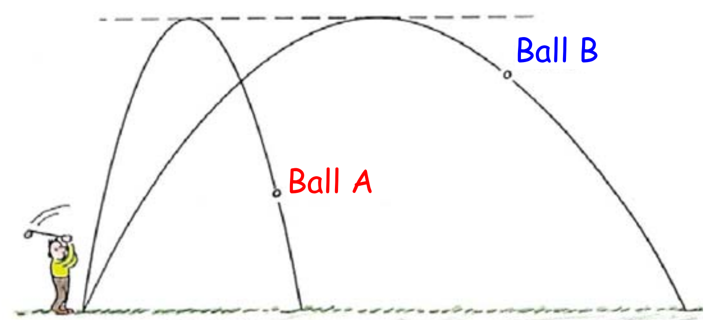


Figure 1

Adapted from Hewitt's Figuring Physics - Physics Today

Question #1.21: Flight Times of Different Parabolas

Does Ball B spend more/ the same/ less time in the air than Ball A?

(a) More time
(b) Same time
(c) Less time

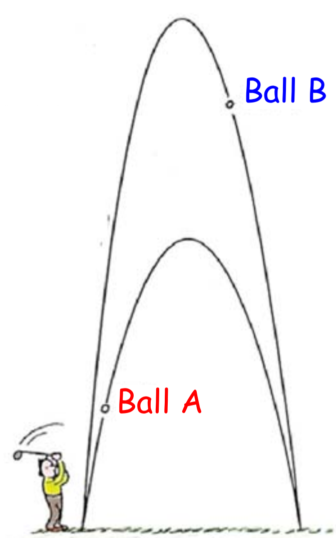


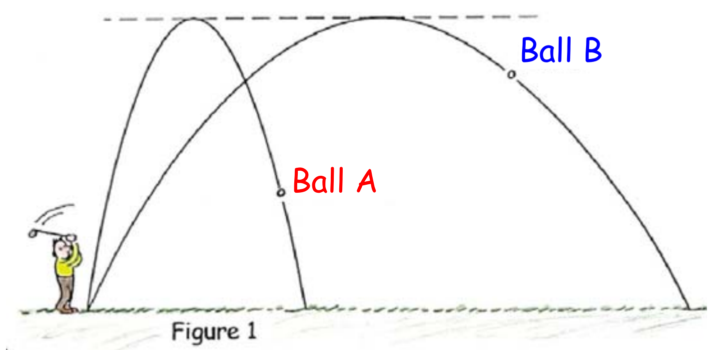
Figure 2

Adapted from Hewitt's Figuring Physics - Physics Today

Question #1.22: Launch Speeds of Parabolas

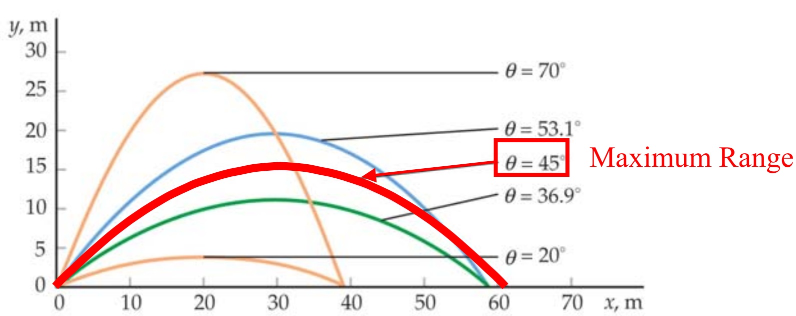
Does Ball B have a larger/ the same/ smaller launch speed than Ball A?

(a) Larger (b) Same (c) Smaller



Adapted from Hewitt's Figuring Physics - Physics Today

Range of Complete Parabolic Path



$$\boxed{x = (v_0 \cos\theta) t}, \quad \text{where } t_{\text{full arc}} = \frac{2v_{0y}}{g} = \frac{2(v_0 \sin\theta)}{g}$$

$$x = (v_0 \cos\theta)\left(\frac{2(v_0 \sin\theta)}{g}\right) = \frac{2(v_0)^2 \cos\theta \sin\theta}{g}$$

$$\boxed{x = \frac{(v_0)^2 \sin 2\theta}{g}} \quad \text{Max. range for } \theta = 45°.$$

Problem #1.7: Range and Velocity of Brick

In a contest, a strong man threw a brick 50 m across a field. Find the velocity of the brick at its highest point. Assume that brick's range was maximized by launching it at 45°. (Use g = 10 m/s^2.)

Solve for v_0 using the range equation.

$$\boxed{x} = \frac{v_0^{\,2} \sin 2\theta}{g} = \frac{v_0^{\,2}}{g} \quad \text{where } \sin 2\theta = \sin 2(45°) = 1$$

$$\boxed{v_0} = \sqrt{xg} = \sqrt{(50\text{ m})(10\text{ m/s}^2)} = \underline{22\text{ m/s}}$$

At its apex, the brick's velocity equals its horizontal component v_{0x}.

$$v_x = v_{x0} = v_0 \cos\theta = (22\text{ m/s})\cos 45°$$

$$\boxed{v_x = 16\text{ m/s}} \text{ or } {\sim}35\text{ mph}$$

Question #1.23: Range of Thrown Balls

Two footballs are thrown from the same point at 30° above the horizontal. Ball B has twice the initial speed of Ball A.

If *Ball A* is caught at distance x_A from the thrower, at what distance x_B will *Ball B* be caught from the thrower?

(a) $x_B = 8x_A$

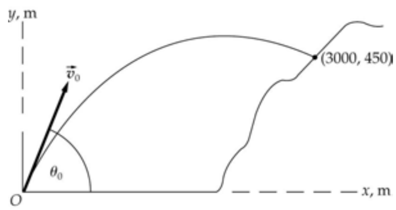
(b) $x_B = 4x_A$

(c) $x_B = 2x_A$

(d) $x_B = x_A$

Problem #1.8: Find Projectile's Initial Velocity

A projectile is fired and lands 20 s later on the side of a hill, at a location 3 km away horizontally and 450 m above its starting point. What are the horizontal and vertical components of its initial velocity?



Horizontal velocity v_x remains constant, so solve for it during flight.

$$\boxed{v_{0x}} = v_x = \frac{\Delta x}{\Delta t} = \frac{3000\text{ m}}{20\text{ s}} = \boxed{150\text{ m/s}}$$

Solve for the vertical velocity v_y from the y-equation.

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \quad \Rightarrow \quad \boxed{v_{0y}} = \frac{(y - y_0) + \frac{1}{2}gt^2}{t}$$

$$\boxed{v_{0y}} = \frac{(450\text{ m} - 0\text{ m}) + \frac{1}{2}(9.8\text{ m/s}^2)(20\text{ s})^2}{(20\text{ s})} = \boxed{120\text{ m/s}}$$

Problem #1.9a: Find Projectile's Time and Range

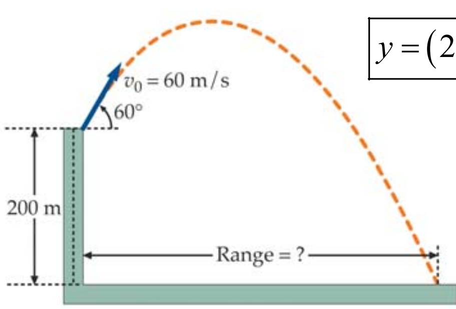
A projectile is fired from the top of a 200-m cliff. Its initial velocity is 60 m/s at 60° above the horizontal. When does the projectile land and what is its range?

Write x- and y-eqns. for projectile motion.

$$\boxed{x} = v_{0x}t = \boxed{(30\text{ m/s})t} \quad \text{and}$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$\boxed{y = (200\text{ m}) + (52\text{ m/s})t - (4.9\text{ m/s}^2)t^2}$$



$$\boxed{v_{0x}} = (60\text{ m/s})\cos 60° = \boxed{30\text{ m/s}}$$

$$\boxed{v_{0y}} = (60\text{ m/s})\sin 60° = \boxed{52\text{ m/s}}$$

Problem #1.9b: Find Projectile's Time and Range

Find flight time t by solving y-equation for $y = 0$ (projectile on ground).

$$y = -\left(4.9\ \text{m/s}^2\right)t^2 + \left(52\ \text{m/s}\right)t + \left(200\ \text{m}\right) = 0$$

$$t = \frac{\left(52\ \text{m/s}\right) \pm \sqrt{\left(52\ \text{m/s}\right)^2 + 4\left(4.9\ \text{m/s}^2\right)\left(200\ \text{m}\right)}}{2\left(4.9\ \text{m/s}^2\right)}$$

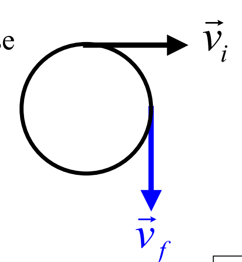
$$\boxed{t} = 5.3 \pm 8.3 = -3.0\ \text{s} \quad \text{or} \quad \boxed{13.6\ \text{s}}$$

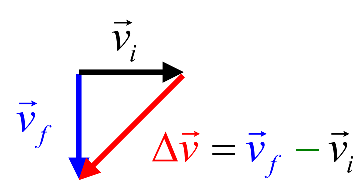
Find range x that projectile traveled during its flight time t.

$$\boxed{x} = \left(30\ \text{m/s}\right)t = \left(30\ \text{m/s}\right)\left(13.6\ \text{s}\right) = \boxed{408\ \text{m}}$$

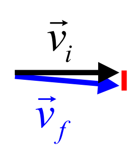
2-D Circular Motion: Direction of Acceleration

Moving Clockwise





$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

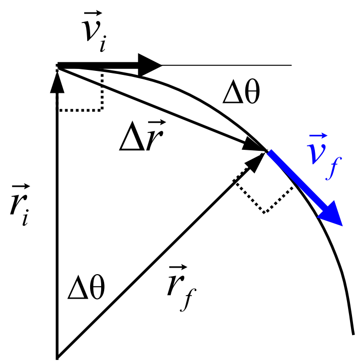


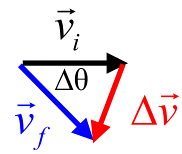
$$\lim_{\Delta t \to 0} \ \vec{a} \perp \vec{v}$$

Circular motion at a constant speed requires an acceleration towards the center of the circle - called centripetal acceleration!

Instantaneous centripetal acceleration is **perpendicular** to velocity.

Circular Motion: Magnitude of Centripetal Acceleration





Similar Triangles: $\dfrac{|\Delta\vec{v}|}{v} = \dfrac{|\Delta\vec{r}|}{r}$

$$\boxed{a_c \text{ or } a_\perp} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v|\Delta\vec{r}|}{r\Delta t} = \boxed{\frac{v^2}{r}}$$

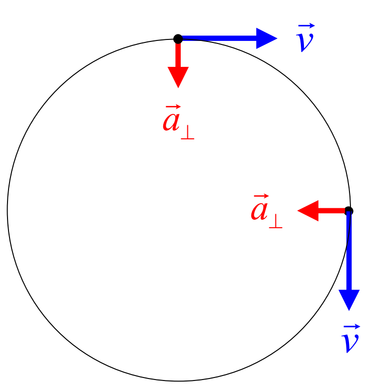
"Perfect" Circular Motion: Directions of v and a

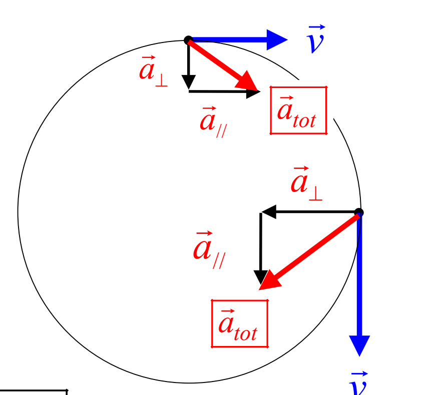
Constant Speed

$\vec{a}_\perp$ "turns" $\vec{v}$

Increasing Speed

$\vec{a}_\perp$ "turns" $\vec{v}$ & $\vec{a}_{//}$ increases $|\vec{v}|$

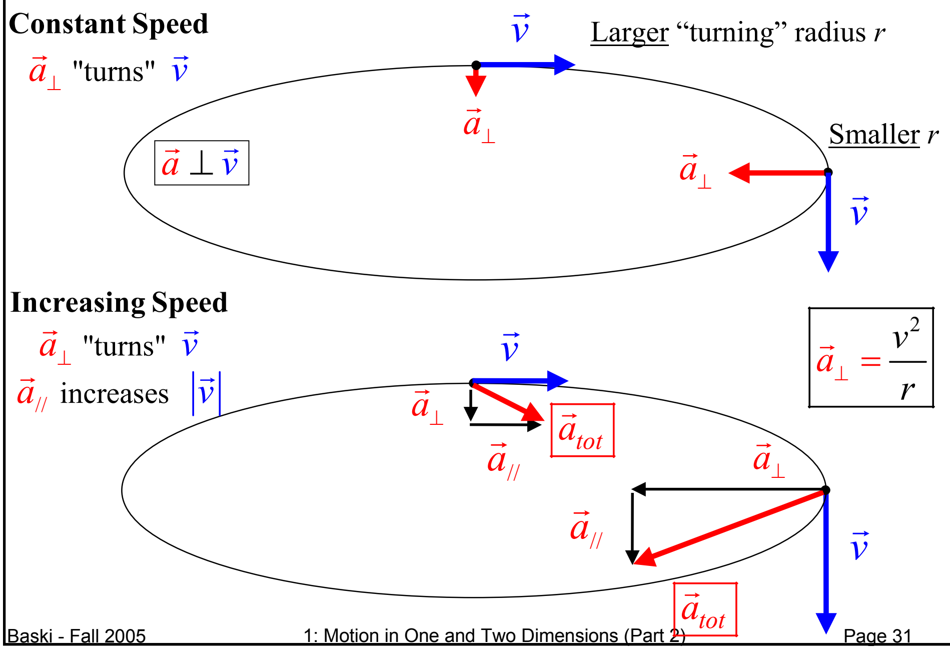




$\boxed{\vec{a} \perp \vec{v}}$

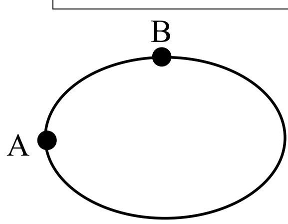
$$\boxed{\vec{a}_\perp = a_c = \frac{v^2}{r}}$$

“General” Circular Motion: v and a for Oval

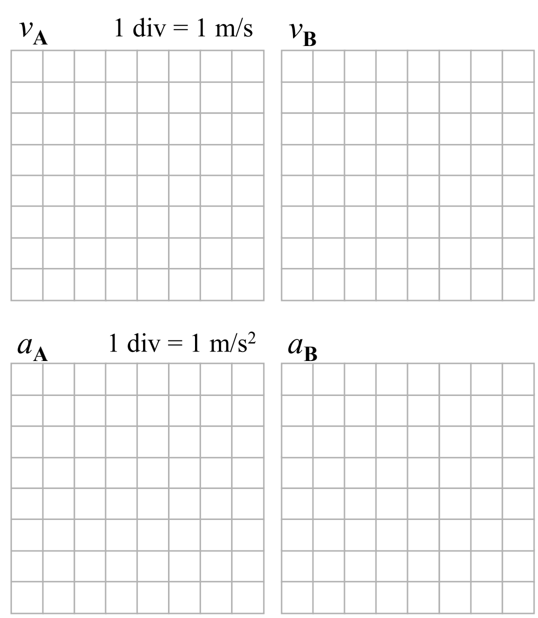


Problem #1.10: v and a for Circular Motion

MUST COMPLETE!!



$v_A = 3$ m/s; $v_B = 5$ m/s
$a = 2$ m/s^2 (**speed up**)
$r_A = 2$ m; $r_B = 4$ m



Question #1.24: Motion for *a* perpendicular to *v*

An instantaneous acceleration that is perpendicular to the motion of an object results in the following:

(a) Object slows down/speeds up AND changes direction

(b) Object only slows down/speeds up

(c) Object only changes direction

(d) Not sufficient information to predict resulting motion of object

Question #1.25: Angle between *a* and *v*

For an object speeding up as it goes around a curve, the angle θ between the acceleration and velocity vectors is:

(a) 90° $(a \perp v)$

(b) 0° $(a \,//\, v)$

(c) $0° < \theta < 90°$

(d) $90° < \theta < 180°$