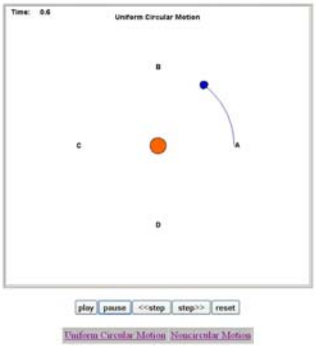
RECITATION HANDOUT #13: Gravitational Orbits

Properties of Circular vs. Elliptical Orbits

There are **two types** of closed orbits for a planetary system: **circular** and **elliptical** (circle = special ellipse). Use http://www.courses.vcu.edu/PHYS-jms/PHYS207/gravity/circ_ellip_motion.htm to answer the following questions about orbital motion for these two cases. The inner circle represents the sun (mass M) and the outer circle represents a planet (mass m) orbiting the sun. *We assume that the sun remains fixed in place.*



Circular Orbit

(a) In the circular orbit, **how long** does it take the planet to go from point A to point B? ____________ s

(b) **How long** does it take the planet to from point B to point C? ____________ s

(c) At **what point** (A,B,C,D, or all the same) is the **kinetic energy** maximum? ________________

(d) At **what point** (A,B,C,D, or all the same) is the **potential energy** maximum? ________________
Remember that the gravitational potential energy equals $-\frac{GMm}{r}$.

(e) How do the **gravitational** force $\frac{GMm}{r^2}$ and **centripetal** force $\frac{mv^2}{r}$ compare at points A and C?

(circle the correct relationship)

Point A: $\frac{GMm}{r_A^2} > \frac{mv^2}{r_A}$, $\frac{GMm}{r_A^2} < \frac{mv^2}{r_A}$, or $\frac{GMm}{r_A^2} = \frac{mv^2}{r_A}$

Point C: $\frac{GMm}{r_C^2} > \frac{mv^2}{r_C}$, $\frac{GMm}{r_C^2} < \frac{mv^2}{r_C}$, or $\frac{GMm}{r_C^2} = \frac{mv^2}{r_C}$

Elliptical Orbit

(a) In the elliptical orbit, **how long** does it take the planet to go from point A to point B? ____________ s

(b) **How long** does it take the planet to from point B to point C? ____________ s

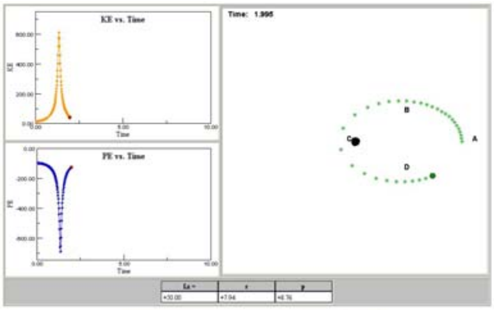
(c) At **what point** (A,B,C,D, or all the same) is the **kinetic energy** maximum? ____________

(d) At **what point** (A,B,C,D, or all the same) is the **potential energy** maximum? ____________
Remember that the potential energy $-\frac{GMm}{r}$ increases (or becomes less negative) as the radius increases. Also, no external force acts on the system, so total energy is conserved.

(e) How do the **gravitational** force $\frac{GMm}{r^2}$ and **centripetal** force $\frac{mv^2}{r}$ compare at points A and C?

Point A: $\frac{GMm}{r_A^2} > \frac{mv^2}{r_A}$, $\frac{GMm}{r_A^2} < \frac{mv^2}{r_A}$, or $\frac{GMm}{r_A^2} = \frac{mv^2}{r_A}$

Point C: $\frac{GMm}{r_C^2} > \frac{mv^2}{r_C}$, $\frac{GMm}{r_C^2} < \frac{mv^2}{r_C}$, or $\frac{GMm}{r_C^2} = \frac{mv^2}{r_C}$



Lz =	r	p
+30.00	+7.94	+6.76

Angular Momentum and Energy of Elliptical Orbits

Use http://www.courses.vcu.edu/PHYS-jms/PHYS207/gravity/ke_pe_orbit.htm to answer the following questions. Assume that the mass of the "planet" is **2 kg** and that the mass of the sun is such that $\boldsymbol{GM_{sun}} = \mathbf{1000}$. *We also assume that the sun remains fixed in place.*

(a) With the initial conditions such that $\boldsymbol{r_o}$ = 10 m and $\boldsymbol{v_{oy}}$ = 5 m/s, what **type of orbit** does the planet make? (circular, elliptical, hyperbolic, parabolic)

(b) What is the **angular momentum** of the planet? Hint: $\boldsymbol{L}_z$ is given on the applet page.

$L_z = \boxed{\qquad}$ kg m^2/s

(c) Is there an **x or y component** of the angular momentum? Why or why not?

(d) In general, does $\boldsymbol{L_z = r\,p}$, where $\boldsymbol{r}$ is the distance of the planet to the sun, and $\boldsymbol{p}$ is the magnitude of the momentum? Why or why not?

(e) Is the angular momentum of the planet **constant** during the orbit? Why or why not?

(f) The **kinetic** energy is **maximum** at point ______________ and **minimum** at point _____________.

(g) The **potential** energy is **maximum** at point ______________ and **minimum** at point _____________.

Conditions for Circular Orbits

(h) Can you create a stable **circular** orbit? With $\boldsymbol{r_o}$ = 10 m, what **velocity** $\boldsymbol{v_{yo}}$ is required to produce a circular orbit? Try different values for v_{yo} in the physlet. Can you justify your answer using the relationship between the gravitational and centripetal forces for a circular orbit?

For $r_0 = 10$ m, $v_{y0} = \boxed{\qquad}$ m/s

(i) If instead you start with $\boldsymbol{r_o}$ = 5 m, what **velocity** $\boldsymbol{v_{yo}}$ is required to produce a circular orbit? How does this velocity compare with the larger radius in part (h)?

For $r_0 = 5$ m, $v_{y0} = \boxed{\qquad}$ m/s

(j) If instead you start with $\boldsymbol{r_o}$ = 40 m, what **velocity** $\boldsymbol{v_{yo}}$ is required to produce a circular orbit? You will need to calculate this value – look at the hint given in part (h).

For $r_0 = 40$ m, $v_{y0} = \boxed{\qquad}$ m/s